

2002年度 基礎数学ワークブック

| | |
|-----|---|
| 著者 | 井上 昌昭 |
| 雑誌名 | 高知工科大学 基礎数学ワークブック |
| 巻 | 2002年度版 |
| 発行年 | 2002 |
| URL | http://hdl.handle.net/10173/248 |

高知工科大学

基礎数学ワークブック

(2002年度版)

Series A

No. 7

解答

< 1 ページ. 指数の復習 >

問 1 の解答

(1) $64^{\frac{1}{3}} = 4$

(2) $7^0 = 1$

(3) $3^{-2} = \frac{1}{9}$

(4) $4^{-\frac{1}{2}} = \frac{1}{2}$

(5) $27^{\frac{4}{3}} = 81$

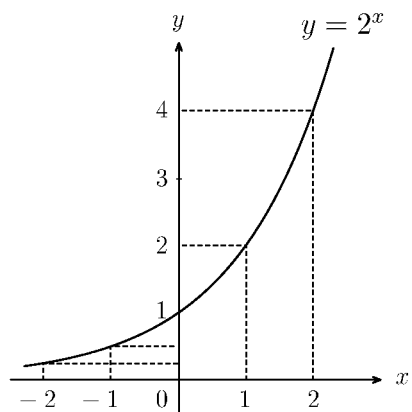
(6) $8^{-\frac{2}{3}} = \frac{1}{4}$

(7) $\left(\frac{1}{25}\right)^{\frac{1}{2}} = \frac{1}{5}$

(8) $(0.0001)^{-0.25} = \left(\frac{1}{10000}\right)^{-\frac{1}{4}} = (10000)^{\frac{1}{4}} = 10$

問 2 の解答

| | | | | | |
|-------|---------------|---------------|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 |
| 2^x | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 |



< 2 ページ. 対数の復習 >

問 1 の解答

(1) $\log_2 8 = 3$

(2) $\log_7 49 = 2$

(3) $\log_3 1 = 0$

(4) $\log_3 \left(\frac{1}{9}\right) = -2$

(5) $\log_{27} 3 = \frac{1}{3}$

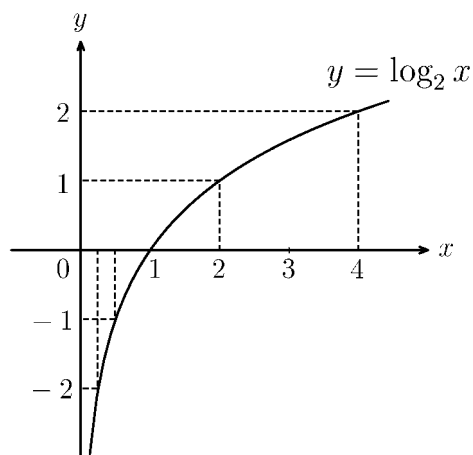
(6) $\log_{32} 8 = \frac{3}{5}$

(7) $\log_{10}(0.01) = -2$

(8) $\log_5(0.2) = \log_5 \left(\frac{1}{5}\right) = -1$

問 2 の解答

| | | | | | |
|------------|---------------|---------------|---|---|---|
| x | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 |
| $\log_2 x$ | -2 | -1 | 0 | 1 | 2 |

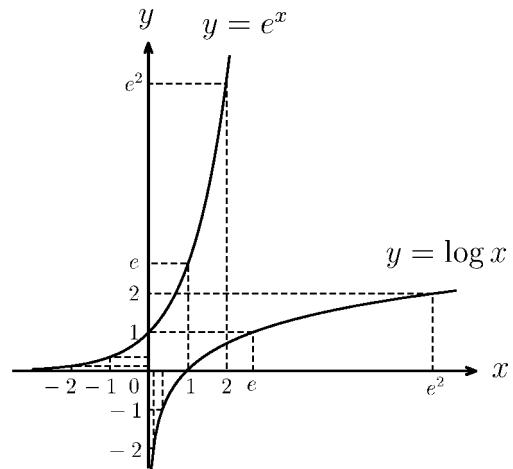


< 3 ページ.eの復習 >

問の解答

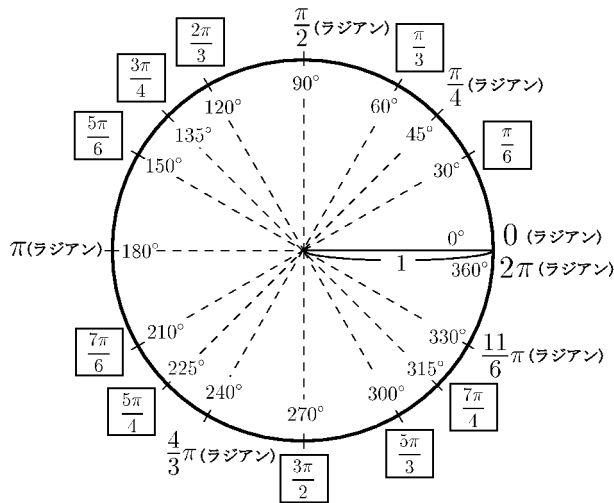
| | | | | | |
|-------|-----------------|---------------|---|-----|-------|
| x | -2 | -1 | 0 | 1 | 2 |
| e^x | $\frac{1}{e^2}$ | $\frac{1}{e}$ | 1 | e | e^2 |

| | | | | | |
|----------|-----------------|---------------|---|-----|-------|
| x | $\frac{1}{e^2}$ | $\frac{1}{e}$ | 1 | e | e^2 |
| $\log x$ | -2 | -1 | 0 | 1 | 2 |



< 4 ページ. 三角関数の復習 1 >

問 1 の解答



問 2 の解答

| | | | | | | | | | | |
|---------------|-----|-----------|----------------------|----------------------|----------------------|-----------------|----------------------|-----------------------|-----------------------|-------------|
| 角度 θ | 度数法 | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
| | 弧度法 | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| $\sin \theta$ | | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| $\cos \theta$ | | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | -1 |
| $\tan \theta$ | | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | X | $-\sqrt{3}$ | -1 | $-\frac{\sqrt{3}}{3}$ | 0 |

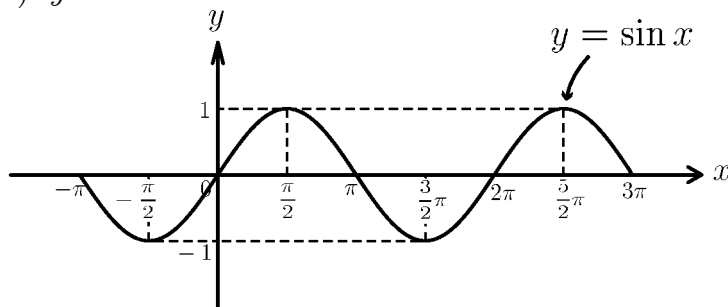
| | | | | | | | | | |
|---------------|-----|-----------------------|-----------------------|-----------------------|------------------|-----------------------|-----------------------|-----------------------|-------------|
| 角度 θ | 度数法 | 210° | 225° | 240° | 270° | 300° | 315° | 330° | 360° |
| | 弧度法 | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | 2π |
| $\sin \theta$ | | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | -1 | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{1}{2}$ | 0 |
| $\cos \theta$ | | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\tan \theta$ | | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | X | $-\sqrt{3}$ | -1 | $-\frac{\sqrt{3}}{3}$ | 0 |

< 5 ページ. 三角関数の復習 2 >

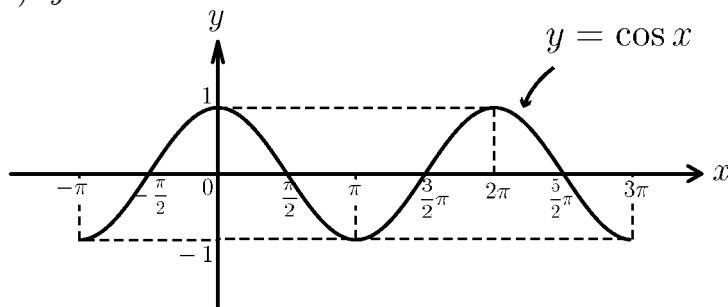
問の解答

| | | | | | | | | | | | | | | | | | | |
|----------|-----|--------------|-----------------------|------------------|-----------------------|-----------|----------------------|-----------------|-----------------------|-------------|-----------------------|------------------|-----------------------|-------------|----------------------|------------------|-----------------------|-------------|
| x | 度数法 | -180° | -135° | -90° | -45° | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° | 405° | 450° | 495° | 540° |
| | 弧度法 | $-\pi$ | $-\frac{3\pi}{4}$ | $-\frac{\pi}{2}$ | $-\frac{\pi}{4}$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | π | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ | 2π | $\frac{9\pi}{4}$ | $\frac{5\pi}{2}$ | $\frac{11\pi}{4}$ | 3π |
| $\sin x$ | | 0 | $-\frac{\sqrt{2}}{2}$ | -1 | $-\frac{\sqrt{2}}{2}$ | 0 | $\frac{\sqrt{2}}{2}$ | 1 | $\frac{\sqrt{2}}{2}$ | 0 | $-\frac{\sqrt{2}}{2}$ | -1 | $-\frac{\sqrt{2}}{2}$ | 0 | $\frac{\sqrt{2}}{2}$ | 1 | $\frac{\sqrt{2}}{2}$ | 0 |
| $\cos x$ | | -1 | $-\frac{\sqrt{2}}{2}$ | 0 | $\frac{\sqrt{2}}{2}$ | 1 | $\frac{\sqrt{2}}{2}$ | 0 | $-\frac{\sqrt{2}}{2}$ | -1 | $-\frac{\sqrt{2}}{2}$ | 0 | $\frac{\sqrt{2}}{2}$ | 1 | $\frac{\sqrt{2}}{2}$ | 0 | $-\frac{\sqrt{2}}{2}$ | -1 |

(1) $y = \sin x$

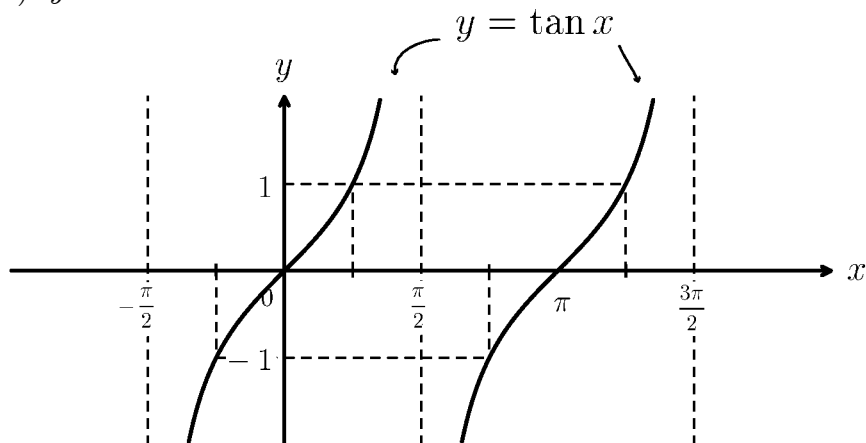


(2) $y = \cos x$



| | | | | | | | | | | | | | | | | | | |
|----------|-----|------------------|------------------|------------------|-----------------------|-----------|----------------------|-----------------|-----------------|-----------------|------------------|------------------|-----------------------|-------------|----------------------|------------------|------------------|------------------|
| x | 度数法 | -90° | -60° | -45° | -30° | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° | 210° | 225° | 240° | 270° |
| | 弧度法 | $-\frac{\pi}{2}$ | $-\frac{\pi}{3}$ | $-\frac{\pi}{4}$ | $-\frac{\pi}{6}$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ |
| $\tan x$ | | \times | $-\sqrt{3}$ | -1 | $-\frac{\sqrt{3}}{3}$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | \times | $-\sqrt{3}$ | -1 | $-\frac{\sqrt{3}}{3}$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | \times |

(3) $y = \tan x$



< 6 ページ. 微分の復習 1 >

問の解答

(1) $(x^3 + x^2)' = 3x^2 + 2x$

(2) $(3x^4 - 2x + 1)' = 12x^3 - 2$

(3) $(\sqrt[3]{x})' = (x^{\frac{1}{3}})' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

(4) $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

(5) $\left(\frac{1}{\sqrt{x}}\right)' = (x^{-\frac{1}{2}})' = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x\sqrt{x}}$

(6) $(\sqrt{x^3})' = (x^{\frac{3}{2}})' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

(7) $\left(\frac{2}{x^3}\right)' = (2x^{-3})' = -6x^{-4} = -\frac{6}{x^4}$

(8) $\left(\frac{x^3 - 2x^2 - 1}{x^2}\right)' = \left(x - 2 - \frac{1}{x^2}\right)' = 1 + \frac{2}{x^3}$

(9) $\left(\frac{x^2 - x}{\sqrt{x}}\right)' = (x^{\frac{3}{2}} - x^{\frac{1}{2}})' = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}} = \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}$

< 7 ページ. 微分の復習 2 >

問の解答

(1) $(x \cos x)' = \cos x - x \sin x$

(2) $(e^x \sin x)' = e^x \sin x + e^x \cos x$

(3) $(e^x \cos x)' = e^x \cos x - e^x \sin x$

(4) $(x + x \log x)' = 1 + \log x + 1 = 2 + \log x$

(5) $\left(\frac{1}{1-x}\right)' = -\frac{(1-x)'}{(1-x)^2} = -\frac{-1}{(1-x)^2} = \frac{1}{(1-x)^2}$

(6) $\left(\frac{x}{1-x}\right)' = \frac{1(1-x) - x(-1)}{(1-x)^2} = \frac{1}{(1-x)^2}$

$$\left(\text{または } \left(\frac{x}{1-x}\right)' = \left(\frac{-(1-x)+1}{1-x}\right)' = \left(-1 + \frac{1}{1-x}\right)' = \left(\frac{1}{1-x}\right)' = \frac{1}{(1-x)^2}\right)$$

(7) $\left(\frac{2}{\sin x}\right)' = -\frac{2x(\sin x)'}{(\sin x)^2} = -\frac{2 \cos x}{\sin^2 x}$

(8) $\left(\frac{1}{e^x}\right)' = -\frac{e^x}{(e^x)^2} = -\frac{1}{e^x}$

$$\left(\text{または } \left(\frac{1}{e^x}\right)' = (e^{-x})' = -e^{-x} = -\frac{1}{e^x}\right)$$

(9) $\left(\frac{\cos x}{\sin x}\right)' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$

< 8 ページ. 微分の復習 3 >

問の解答

(1) $(\sin(1-x))' = -\cos(1-x)$

(2) $(\cos(x^2+2))' = -2x \sin(x^2+2)$

(3) $(e^{x^4})' = 4x^3 e^{x^4}$

(4) $((2x+1)^6)' = 12(2x+1)^5$

(5) $(\sqrt{2x-5})' = \frac{1}{\sqrt{2x-5}}$

(6) $(\cos(2x) + e^{x-1})' = -2 \sin(2x) + e^{x-1}$

(7) $(e^{2x} \sin(2x))' = 2e^{2x} \sin(2x) + 2e^{2x} \cos(2x)$

(8) $(e^{4x} \sin(3x))' = 4e^{4x} \sin(3x) + 3e^{4x} \cos(3x)$

(9) $(\cos(-3x) \sin(2x))' = 3 \sin(-3x) \sin(2x) + 2 \cos(-3x) \cos(2x)$

$$\left(\begin{aligned} \text{または } (\cos(-3x) \sin(2x))' &= (\cos(3x) \sin(2x))' \\ &= -3 \sin(3x) \sin(2x) + 2 \cos(3x) \cos(2x) \end{aligned} \right)$$

< 9 ページ. 接線の傾き 1 >

問の解答

$$f'(x) = -3x^2 + 6x$$

$x = -1$ における接線の傾き

$$f'(-1) = -3 - 6 = -9$$

$x = 0$ における接線の傾き

$$f'(0) = 0$$

$x = 1$ における接線の傾き

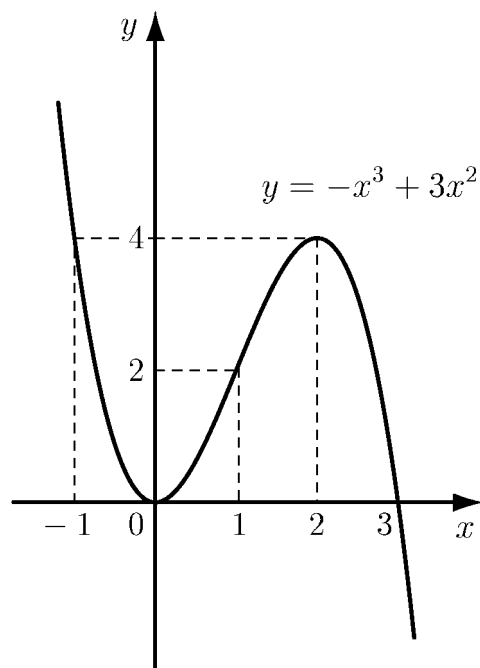
$$f'(1) = -3 + 6 = 3$$

$x = 2$ における接線の傾き

$$f'(2) = -12 + 12 = 0$$

$x = 3$ における接線の傾き

$$f'(3) = -27 + 18 = -9$$



< 10 ページ. 接線の傾き 2 >

問の解答

(1) $y = e^x$ の $x = 1$ における接線の傾き $= e^1 = e$

(2) $y = e^x$ の $x = -1$ における接線の傾き $= e^{-1} = \frac{1}{e}$

(3) $f(x) = \sin x$ の $x = 0$ における接線の傾き $= \cos 0 = 1$

(4) $f(x) = \sin x$ の $x = \pi$ における接線の傾き $= \cos \pi = -1$

(5) $f(x) = \cos x$ の $x = 0$ における接線の傾き $= -\sin 0 = 0$

(6) $f(x) = \cos x$ の $x = \frac{\pi}{2}$ における接線の傾き $= -\sin\left(\frac{\pi}{2}\right) = -1$

(7) $f(x) = \log x$ の $x = 1$ における接線の傾き $= \frac{1}{1} = 1$

(8) $f(x) = \log x$ の $x = 2$ における接線の傾き $= \frac{1}{2}$

< 11 ページ. 接線の方程式 1 >

問 1 の解答

$$y = m(x - a) + b$$

問 2 の解答

$$y' = 3x^2 - 2x + 1$$

$$x = 1 \text{ のとき } y' = 3 - 2 + 1 = 2 \text{ (傾き 2)}$$

$$y = 2(x - 1) + 1 = 2x - 1$$

$$\underline{\text{(答) } y = 2x - 1}$$

問 3 の解答

$$y = f'(a)(x - a) + b$$

< 12 ページ. 接線の方程式 2 >

問の解答

(1) $y = e^x$ の $x = 0$ における接線

$$y' = e^x, x = 0 \text{ のとき } y = e^0 = 1, y' = e^0 = 1$$

$$\underline{\text{(答) } y = x + 1}$$

(2) $y = \log x$ の $x = 1$ における接線

$$y' = \frac{1}{x}, x = 1 \text{ のとき } y = \log 1 = 0, y' = \frac{1}{1} = 1$$

$$\underline{\text{(答) } y = x - 1}$$

(3) $y = \sin x$ の $x = 0$ における接線

$$y' = \cos x, x = 0 \text{ のとき } y = \sin 0 = 0, y' = \cos 0 = 1$$

$$\underline{\text{(答) } y = x}$$

(4) $y = \sqrt{x}$ の $x = 4$ における接線

$$y' = \frac{1}{2\sqrt{x}}, x = 4 \text{ のとき } y = \sqrt{4} = 2, y' = \frac{1}{2\sqrt{4}} = \frac{1}{4}, y = \frac{1}{4}(x - 4) + 2$$

$$\underline{\text{(答) } y = \frac{1}{4}x + 1}$$

(5) $y = \frac{1}{x}$ の $x = 1$ における接線

$$y' = -\frac{1}{x^2}, x = 1 \text{ のとき } y = \frac{1}{1} = 1, y' = -\frac{1}{1^2} = -1, y = -1(x - 1) + 1$$

$$\underline{\text{(答) } y = -x + 2}$$

< 13 ページ. 関数の一次近似 >

問の解答

$$f(x) = \sqrt[4]{x} \text{ とおくと } f(a) = \sqrt[4]{a} ,$$

$$f'(x) = (\sqrt[4]{x})' = \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4\sqrt[4]{x^3}} \text{ より } f'(a) = \frac{1}{4\sqrt[4]{a^3}}$$

$$x \doteq a \text{ のとき } \sqrt[4]{x} \doteq \sqrt[4]{a} + \frac{1}{4\sqrt[4]{a^3}}(x - a)$$

ここで $a = 1$, $x = 1.1$ とおけば

$$\sqrt[4]{1.1} \doteq \sqrt[4]{1} + \frac{1}{4\sqrt[4]{1^3}}(1.1 - 1) = 1 + \frac{1}{4} \times 0.1 = 1 + \frac{1}{40}$$

< 14 ページ. 高階導関数 >

問 1 の解答

(1) $f(x) = x^2 - 3x + 2$

$$f'(x) = 2x - 3$$

$$f''(x) = 2$$

(2) $f(x) = \sin x$

$$f'(x) = \cos x$$

$$f''(x) = -\sin x$$

(3) $f(x) = \log x$

$$f'(x) = \frac{1}{x}$$

$$f''(x) = -\frac{1}{x^2}$$

問 2 の解答

(1) $f(x) = x^5 - x^3 + x$

$$f'(x) = 5x^4 - 3x^2 + 1$$

$$f''(x) = 20x^3 - 6x$$

$$f'''(x) = 60x^2 - 6$$

(2) $f(x) = \cos x$

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x$$

$$f'''(x) = \sin x$$

(3) $f(x) = x \log x$

$$f'(x) = \log x + 1$$

$$f''(x) = \frac{1}{x}$$

$$f'''(x) = -\frac{1}{x^2}$$

(4) $f(x) = e^{2x}$

$$f'(x) = 2e^{2x}$$

$$f''(x) = 4e^{2x}$$

$$f'''(x) = 8e^{2x}$$

< 15 ページ. グラフの凹凸 1 >

問の解答

(1) $y = x^3 + 3x^2 - 9x$

$y'' = 6x + 6$

| | | | |
|-------|-----|----|-----|
| x | ... | -1 | ... |
| y'' | - | 0 | + |
| y | 凸 | 11 | 凹 |

(2) $y = -x^4 + 2x^3 + 12x^2 - 10$

$y'' = -12x^2 + 12x + 24 = -12(x^2 - x - 2) = -12(x - 2)(x + 1)$

| | | | | | |
|-------|-----|----|-----|----|-----|
| x | ... | -1 | ... | 2 | ... |
| y'' | - | 0 | + | 0 | - |
| y | 凸 | -1 | 凹 | 38 | 凸 |

< 16 ページ. グラフの凹凸 2 >

問の解答

(1) $y = x^2 - 5x + 6$

| | | | |
|-------|-----|----------------|-----|
| x | ... | $\frac{5}{2}$ | ... |
| y' | - | 0 | + |
| y'' | + | + | + |
| y | ↘ | $-\frac{1}{4}$ | ↗ |

(2) $y = -x^2 + 4x - 5$

| | | | |
|-------|-----|----|-----|
| x | ... | 2 | ... |
| y' | + | 0 | - |
| y'' | - | - | - |
| y | ↗ | -1 | ↘ |

< 17 ページ. グラフの凹凸 3 >

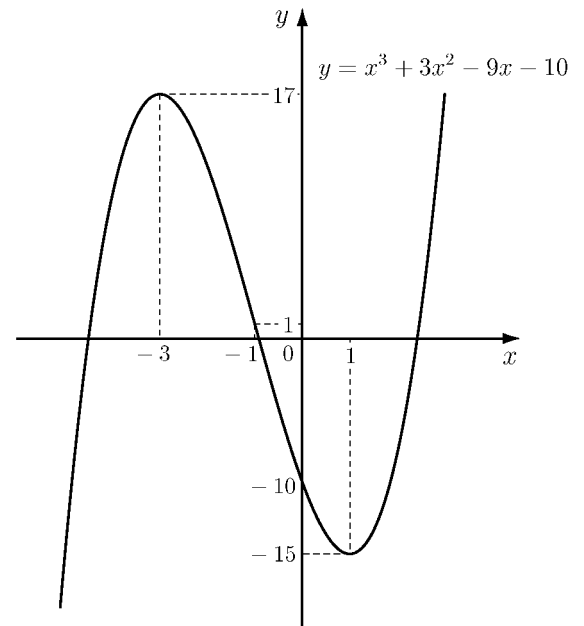
問の解答

(解)

$$\begin{aligned} y' &= 3x^2 + 6x - 9 \\ &= 3(x^2 + 2x - 3) \\ &= 3(x + 3)(x - 1) \end{aligned}$$

$$y'' = 6x + 6 = 6(x + 1)$$

| | | | | | | | |
|-------|-----|----|-----|----|-----|-----|-----|
| x | ... | -3 | ... | -1 | ... | 1 | ... |
| y' | + | 0 | - | - | - | 0 | + |
| y'' | - | - | - | 0 | + | + | + |
| y | ↖ | 17 | ↘ | 1 | ↙ | -15 | ↗ |



< 18 ページ. 微分係数と極限值 >

問の解答

$$(1) \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = f'(0) = e^0 = 1$$

$$(2) \lim_{x \rightarrow 2} \frac{x^6 - 64}{x - 2} = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = f'(2) = 6 \times 2^5 = 192$$

$$(3) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{f(x) - f(\frac{\pi}{2})}{x - \frac{\pi}{2}} = f' \left(\frac{\pi}{2} \right) = \cos \left(\frac{\pi}{2} \right) = 0$$

< 19 ページ. ロピタルの定理 1 >

問の解答

$$(1) \lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2} = \lim_{x \rightarrow 2} \frac{(x^4 - 16)'}{(x - 2)'} = \lim_{x \rightarrow 2} \frac{4x^3}{1} = 4 \times 2^3 = 32$$

$$(2) \lim_{x \rightarrow 1} \frac{e^x - e}{x - 1} = \lim_{x \rightarrow 1} \frac{(e^x - e)'}{(x - 1)'} = \lim_{x \rightarrow 1} \frac{e^x}{1} = e' = e$$

$$(3) \lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{(\sin x)'}{(x)'} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \cos 0 = 1$$

< 20 ページ. ロピタルの定理 2 >

問の解答

$$(1) \lim_{x \rightarrow 1} \frac{x^5 - 1 - 5(x-1)}{(x-1)^2} = \lim_{x \rightarrow 1} \frac{5x^4 - 5}{2(x-1)} = \lim_{x \rightarrow 1} \frac{20x^3}{2} = \frac{20}{2} = 10$$

$$(2) \lim_{x \rightarrow 2} \frac{x^5 - 2^5 - 5 \times 2^4(x-2)}{(x-2)^2} = \lim_{x \rightarrow 2} \frac{5x^4 - 5 \times 2^4}{2(x-2)} = \lim_{x \rightarrow 2} \frac{20x^3}{2} = 10 \times 2^3 = 80$$

$$(3) \lim_{x \rightarrow 1} \frac{x^4 - 1 - 4(x-1) - 6(x-1)^2}{(x-1)^3} = \lim_{x \rightarrow 1} \frac{4x^3 - 4 + 12(x-1)}{3(x-1)^2}$$
$$= \lim_{x \rightarrow 1} \frac{12x^2 - 12}{6(x-1)} = \lim_{x \rightarrow 1} \frac{24x}{6} = \frac{24}{6} = 4$$

$$(4) \lim_{x \rightarrow 1} \frac{x^5 - 1 - 5(x-1) - 10(x-1)^2 - 10(x-1)^3}{(x-1)^4}$$
$$= \lim_{x \rightarrow 1} \frac{5x^4 - 5 - 20(x-1) - 30(x-1)^2}{4(x-1)^3}$$
$$= \lim_{x \rightarrow 1} \frac{20x^3 - 20 - 60(x-1)}{12(x-1)^2} = \lim_{x \rightarrow 1} \frac{60x^2 - 60}{24(x-1)} = \lim_{x \rightarrow 1} \frac{120x}{24} = \frac{120}{24} = \frac{10}{2} = 5$$

< 21 ページ. 微分記号 1 >

問の解答

$$(1) \frac{d}{dx}\{a^3 + b^4 + c^2\} = 0$$

$$(2) \frac{d}{dx}\{a^3 + b^4x + c^5x^2\} = b^4 + 2c^5x$$

$$(3) \frac{d}{dx}\{(a + b)^3 - c^4\} = 0$$

$$(4) \frac{d}{dx}\{(a - b)^2x - c^3\} = (a - b)^2$$

$$(5) \frac{d}{dx}\{a^4(x - b)\} = a^4$$

$$(6) \frac{d}{dx}\{a^3(x + c)^2\} = 2a^3(x + c)$$

$$(7) \frac{d}{dx}\{(ax + b)^4\} = 4a(ax + b)^3$$

$$(8) \frac{d}{dx}\{(x - a)^5\} = 5(x - a)^4$$

$$(9) \frac{d}{dx}\{a^3(x - a)^3\} = 3a^3(x - a)^2$$

$$(10) \frac{d}{dx}\{4a^3(x - b)^4\} = 16a^3(x - b)^3$$

$$(11) \frac{d}{dx}\{x^2 - a^2 - 2a(x - a)\} = 2x - 2a$$

$$(12) \frac{d}{dx}\{x^3 - a^3 - 3a(x - a) - 6a(x - a)^2\} = 3x^2 - 3a^2 - 12a(x - a)$$

< 22 ページ. 微分記号 2 >

問 1 の解答

$$(1) \frac{d}{dt}(4t^3 + 5t^2 - 2t + 3) = 12t^2 + 10t - 2$$

$$(2) \frac{d}{dy}(5y^6 - 7y^3 + 8y^4 - 4) = 30y^5 - 21y^2 + 32y^3$$

$$(3) \frac{d}{dt}\{(t + 4)^5\} = 5(t + 4)^4$$

$$(4) \frac{d}{dy}\{(3y + 1)^6\} = 18(3y + 1)^5$$

$$(5) \frac{d}{dt}\{10(t - 5)^6\} = 60(t - 5)^5$$

$$(6) \frac{d}{dy}\{15(y - 4)^8\} = 120(y - 4)^7$$

問 2 の解答

$$(1) \frac{d}{dt}\{(a - b)^2t - c\} = (a - b)^2$$

$$(2) \frac{d}{dy}\{a^4(y - b)\} = a^4$$

$$(3) \frac{d}{dt}\{(at + b)^2\} = 2a(at + b)$$

$$(4) \frac{d}{dy}\{(ay - b)^3\} = 3a(ay - b)^2$$

$$(5) \frac{d}{dt}\{a^4(t - 1)^2\} = 2a^4(t - 1)$$

$$(6) \frac{d}{dy}\{a^5(y - b)^3\} = 3a^5(y - b)^2$$

$$(7) \frac{d}{dt}\{a^5(t - a)^4\} = 4a^5(t - a)^3$$

$$(8) \frac{d}{dy}\{3a^2(y + a)^5\} = 15a^2(y + a)^4$$

< 23 ページ. ロピタルの定理 3 >

問の解答

$$(1) \lim_{x \rightarrow a} \frac{x^2 - a^2 - 2a(x - a)}{(x - a)^2} = \lim_{x \rightarrow a} \frac{2x - 2a}{2(x - a)} = 1$$

$$(2) \lim_{x \rightarrow a} \frac{x^4 - a^4 - 4a^3(x - a)}{(x - a)^2} = \lim_{x \rightarrow a} \frac{4x^3 - 4a^3}{2(x - a)} = \lim_{x \rightarrow a} \frac{12x^2}{2} = 6a^2$$

$$(3) \lim_{x \rightarrow a} \frac{x^5 - a^5 - 5a^4(x - a)}{(x - a)^2} = \lim_{x \rightarrow a} \frac{5x^4 - 5a^4}{2(x - a)} = \lim_{x \rightarrow a} \frac{20a^3}{2} = 10a^3$$

< 24 ページ. ロピタルの定理 4 >

問の解答

$$(1) \lim_{y \rightarrow a} \frac{\log y - \log a}{y - a} = \lim_{y \rightarrow a} \frac{\frac{1}{y}}{1} = \frac{1}{a}$$

$$(2) \lim_{t \rightarrow b} \frac{\cos t - \cos b}{t - b} = \lim_{t \rightarrow b} \frac{-\sin t}{1} = -\sin b$$

$$(3) \lim_{\beta \rightarrow a} \frac{a \sin(b\beta) - \beta \sin(ab)}{a^3 - a\beta^2} = \lim_{\beta \rightarrow a} \frac{ab \cos(b\beta) - \sin(ab)}{-2a\beta} = \frac{ab \cos(ab) - \sin(ab)}{-2a^2}$$

< 25 ページ. ロピタルの定理 5 >

問の解答

$$(1) \lim_{x \rightarrow a} \frac{x^4 - a^4 - 4a^3(x-a) - 6a^2(x-a)^2}{(x-a)^3} = \lim_{x \rightarrow a} \frac{4x^3 - 4a^3 - 12a^2(x-a)}{3(x-a)^2}$$

$$= \lim_{x \rightarrow a} \frac{12x^2 - 12a^2}{6(x-a)} = \lim_{x \rightarrow a} \frac{24x}{6} = 4a$$

$$(2) \lim_{x \rightarrow a} \frac{x^6 - a^6 - 6a^5(x-a) - 15a^4(x-a)^2}{(x-a)^3} = \lim_{x \rightarrow a} \frac{6x^5 - 6a^5 - 30a^4(x-a)}{3(x-a)^2}$$

$$= \lim_{x \rightarrow a} \frac{30x^4 - 30a^4}{6(x-a)} = \lim_{x \rightarrow a} \frac{120x^3}{6} = 20a^3$$

$$(3) \lim_{x \rightarrow a} \frac{x^7 - a^7 - 7a^6(x-a) - 21a^5(x-a)^2}{(x-a)^3} = \lim_{x \rightarrow a} \frac{7x^6 - 7a^6 - 42a^5(x-a)}{3(x-a)^2}$$

$$= \lim_{x \rightarrow a} \frac{42x^5 - 42a^5}{6(x-a)} = \lim_{x \rightarrow a} \frac{210x^4}{6} = 35a^4$$

< 26 ページ. ロピタルの定理 6 >

問の解答

$$\begin{aligned}(1) \quad & \lim_{x \rightarrow a} \frac{x^5 - a^5 - 5a^4(x-a) - 10a^3(x-a)^2 - 10a^2(x-a)^3}{(x-a)^4} \\ &= \lim_{x \rightarrow a} \frac{5x^4 - 5a^4 - 20a^3(x-a) - 30a^2(x-a)^2}{4(x-a)^3} \\ &= \lim_{x \rightarrow a} \frac{20x^3 - 20a^3 - 60a^2(x-a)}{12(x-a)^2} = \lim_{x \rightarrow a} \frac{60x^2 - 60a^2}{24(x-a)} = \lim_{x \rightarrow a} \frac{120x}{24} = 5a\end{aligned}$$

$$\begin{aligned}(2) \quad & \lim_{x \rightarrow a} \frac{x^7 - a^7 - 7a^6(x-a) - 21a^5(x-a)^2 - 35a^4(x-a)^3}{(x-a)^4} \\ &= \lim_{x \rightarrow a} \frac{7x^6 - 7a^6 - 42a^5(x-a) - 105a^4(x-a)^2}{4(x-a)^3} \\ &= \lim_{x \rightarrow a} \frac{42x^5 - 42a^5 - 210a^4(x-a)}{12(x-a)^2} = \lim_{x \rightarrow a} \frac{210x^4 - 210a^4}{24(x-a)} \\ &= \lim_{x \rightarrow a} \frac{840x^3}{24} = \frac{840}{24}a^3 = 35a^3\end{aligned}$$

< 27 ページ. 関数の高次近似 1 >

問の解答

$$(1) \quad \frac{d}{dx}\{f(x) - f(a) - f'(a)(x - a)\} = f'(x) - f'(a)$$

$$(2) \quad \frac{d}{dx}\{f'(x) - f'(a) - f''(a)(x - a)\} = f''(x) - f''(a)$$

$$(3) \quad \frac{d}{dx}\{f(x) - f(a) - f'(a)(x - a) - \frac{1}{2}f''(a)(x - a)^2\} \\ = f'(x) - f'(a) - f''(a)(x - a)$$

$$(4) \quad \frac{d}{dx}\{f'(x) - f'(a) - f''(a)(x - a) - \frac{1}{2}f'''(a)(x - a)^2\} \\ = f''(x) - f''(a) - f'''(a)(x - a)$$

$$(5) \quad \frac{d}{dx}\{f(x) - f(a) - f'(a)(x - a) - \frac{1}{2}f''(a)(x - a)^2 - \frac{1}{6}f'''(a)(x - a)^3\} \\ = f'(x) - f'(a) - f''(a)(x - a) - \frac{1}{2}f'''(a)(x - a)^2$$

< 28 ページ. 関数の高次近似 2 >

問の解答

$$\begin{aligned} (1) \quad & \lim_{x \rightarrow a} \frac{f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2}{(x-a)^3} \\ &= \lim_{x \rightarrow a} \frac{f'(x) - f'(a) - f''(a)(x-a)}{3(x-a)^2} \\ &= \lim_{x \rightarrow a} \frac{f''(x) - f''(a)}{6(x-a)} = \lim_{x \rightarrow a} \frac{f'''(x)}{6} = \frac{f'''(a)}{6} \end{aligned}$$

$$\begin{aligned} (2) \quad & \lim_{x \rightarrow a} \frac{f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2 - \frac{1}{6}f'''(a)(x-a)^3}{(x-a)^4} \\ &= \lim_{x \rightarrow a} \frac{f'(x) - f'(a) - f''(a)(x-a) - \frac{1}{2}f'''(a)(x-a)^2}{4(x-a)^3} \\ &= \lim_{x \rightarrow a} \frac{f''(x) - f''(a) - f'''(a)(x-a)}{12(x-a)^2} \\ &= \lim_{x \rightarrow a} \frac{f'''(x) - f'''(a)}{24(x-a)} = \lim_{x \rightarrow a} \frac{f''''(x)}{24} = \frac{1}{24}f''''(a) \end{aligned}$$

< 29 ページ. 関数の高次近似 3 >

問の解答

$$f(x) \doteq f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{6}f'''(a)(x-a)^3 + \frac{1}{24}f''''(a)(x-a)^4$$

< 30 ページ. 高階微分係数 >

問の解答

$$(1) f^{(4)}(x) = e^x, f^{(4)}(0) = e^0 = 1$$

$$(2) f^{(n)}(x) = e^x, f^{(n)}(0) = e^0 = 1$$

$$(3) f^{(1)}(x) = \cos x, f^{(2)}(x) = -\sin x, f^{(3)}(x) = -\cos x, f^{(4)}(x) = \sin x$$

$$f^{(5)}(x) = \cos x, f^{(6)}(x) = -\sin x, f^{(7)}(x) = -\cos x, f^{(8)}(x) = \sin x$$

$$f^{(1)}(0) = 1, f^{(2)}(0) = 0, f^{(3)}(0) = -1, f^{(4)}(0) = 0$$

$$f^{(5)}(0) = 1, f^{(6)}(0) = 0, f^{(7)}(0) = -1, f^{(8)}(0) = 0$$

< 31 ページ. 関数の n 次近似 1 >

問の解答

$$f(x) \doteq f(a) + \frac{1}{1!} f^{(1)}(a)(x-a) + \frac{1}{2!} f^{(2)}(a)(x-a)^2 + \frac{1}{3!} f^{(3)}(a)(x-a)^3 + \frac{1}{4!} f^{(4)}(a)(x-a)^4$$

< 32 ページ. 関数の n 次近似 2 >

問の解答

$$e^x \doteq e^a + e^a(x-a) + \frac{e^a}{2!}(x-a)^2 + \frac{e^a}{3!}(x-a)^3 + \cdots + \frac{e^a}{n!}(x-a)^n$$

< 33 ページ. テーラー展開 >

問 1 の解答

$$e^x = e^a + e^a(x-a) + \frac{1}{2!}e^a(x-a)^2 + \frac{1}{3!}e^a(x-a)^3 + \frac{1}{4!}e^a(x-a)^4 + \cdots + \frac{1}{n!}e^a(x-a)^n + \cdots$$

問 2 の解答

$$(1) e^x = e + e(x-1) + \frac{e}{2!}(x-1)^2 + \frac{e}{3!}(x-1)^3 + \frac{e}{4!}(x-1)^4 + \cdots + \frac{e}{n!}(x-1)^n + \cdots$$

$$(2) e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$$

< 34 ページ. マクローリン展開 1 >

問の解答

$$f(0) = \sin 0 = 0, f^{(1)}(0) = 1, f^{(2)}(0) = 0, f^{(3)}(0) = -1,$$

$$f^{(4)}(0) = 0, f^{(5)}(0) = 1, f^{(6)}(0) = 0, f^{(7)}(0) = -1, f^{(8)}(0) = 0$$

$$\begin{aligned} \sin x &= 0 + \frac{1}{1!} \times 1 \times x + \frac{1}{2!} \times 0 \times x^2 + \frac{1}{3!} \times (-1) \times x^3 + \frac{1}{4!} \times 0 \times x^4 \\ &\quad + \frac{1}{5!} \times 1 \times x^5 + \frac{1}{6!} \times 0 \times x^6 + \frac{1}{7!} \times (-1) \times x^7 + \frac{1}{8!} \times 0 \times x^8 + \dots \\ &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11} + \dots \end{aligned}$$

< 36 ページ. マクローリン展開 3 >

問の解答

$$e \doteq 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = 2 + \frac{17}{24} = 2.708333\dots$$

< 37 ページ. 実数 >

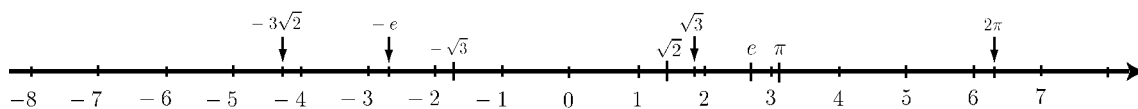
問 1 の解答

(1) $\frac{5}{8} = 0.625$

(2) $\frac{1}{15} = 0.0666\dots = 0.0\dot{6}$

(3) $\frac{3}{11} = 0.27272727\dots = 0.\dot{2}\dot{7}$

問 2 の解答



< 38 ページ. 虚数の導入 1 >

問の解答

(1) $x = \pm 4i$

(2) $x = \pm \frac{5}{2}i$

(3) $x = \pm \frac{\sqrt{3}}{3}i$

< 39 ページ. 虚数の導入 2 >

問の解答

$$(1) \quad x = \frac{1}{2} \pm \sqrt{3}i$$

$$(2) \quad x = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm \sqrt{16}i}{2} = 3 \pm 2i$$

$$(3) \quad x = a \pm \frac{c}{b}i$$

< 40 ページ. 複素数の定義 >

問の解答

$$(1) \quad a = \frac{1}{2}, \quad b = \frac{3}{2}$$

$$(2) \quad a = 0, \quad b = \frac{1 - \sqrt{2}}{3}$$