

2002年度 基礎数学ワークブック

著者	井上 昌昭
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高知工科大学
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Series A

No. 6

解答

< 1 ページ. 部分積分法 1 >

問の解答

$$\int f(x) \times g'(x) dx = f(x) \times g(x) - \int f'(x) \times g(x) dx$$

< 2 ページ. 部分積分法 2 >

問の解答

$$\begin{aligned}(1) \quad \int (3x - 2) \sin x dx &= \int (3x - 2) \times (-\cos x)' dx \\ &= -(3x - 2) \cos x - \int 3(-\cos x) dx \\ &= -(3x - 2) \cos x + 3 \sin x + C\end{aligned}$$

$$(2) \quad \int x e^x dx = \int x \times (e^x)' dx = x e^x - \int 1 \times e^x dx = x e^x - e^x + C$$

$$\begin{aligned}(3) \quad \int (x^2 + 1) \cos x dx &= \int (x^2 + 1) (\sin x)' dx = (x^2 + 1) \sin x - \int 2x \sin x dx \\ &\left(\int 2x \sin x dx = 2x(-\cos x) - \int 2(-\cos x) dx = -2x \cos x + 2 \sin x + C \right) \\ \text{よって } \int (x^2 + 1) \cos x dx &= (x^2 + 1) \sin x + 2x \cos x - 2 \sin x + C\end{aligned}$$

$$\begin{aligned}(4) \quad \int (\log x) \times x dx &= \int (\log x) \times \left(\frac{x^2}{2}\right)' dx = (\log x) \times \frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2} dx \\ &= \frac{x^2}{2} \log x - \frac{1}{4} x^2 + C\end{aligned}$$

< 3 ページ. 三角関数の不定積分 >

問の解答

$$(1) \int \sin^2 x dx = \int \left\{ \frac{1}{2} - \frac{1}{2} \cos(2x) \right\} dx = \frac{1}{2}x - \frac{1}{4} \sin(2x) + C$$

$$(2) \int \cos(3x) \cos(2x) dx = \int \left\{ \frac{1}{2} \cos(5x) + \frac{1}{2} \cos x \right\} dx = \frac{1}{10} \sin(5x) + \frac{1}{2} \sin x + C$$

$$(3) \int \sin(4x) \sin x dx = \int \left\{ \frac{1}{2} \cos(3x) - \frac{1}{2} \cos(5x) \right\} dx = \frac{1}{6} \sin(3x) - \frac{1}{10} \sin(5x) + C$$

< 4 ページ. 不定積分の検証 >

問の解答

$$(1) \left(\frac{1}{4}(x^4 - 1)^4 \right)' = \frac{1}{4} \times 4(x^4 - 1)^3 \times (4x^3) = 4x^3(x^4 - 1)^3 \text{ より正しくない。}$$

$$(2) \left(\frac{1}{2} \log |x^2 - 1| \right)' = \frac{1}{2} \times \frac{(x^2 - 1)'}{x^2 - 1} = \frac{1}{2} \times \frac{2x}{x^2 - 1} = \frac{x}{x^2 - 1} \text{ より正しい。}$$

$$(3) (x^2 e^x - 2x e^x + 2e^x)' = 2x e^x + x^2 e^x - 2e^x - 2x e^x + 2e^x = x^2 e^x \text{ より正しい。}$$

< 5 ページ. 数列の類推 >

問 1 の解答

$$a_1 = 1, a_2 = 5, a_3 = 14, a_4 = 30, a_5 = 55,$$

$$b_1 = 2, b_2 = 3, b_3 = 4, b_4 = 5, b_5 = 6,$$

$$b_n = n + 1,$$

問 2 の解答

$$a_1 = 1, a_2 = 3, a_3 = 6, a_4 = 10, a_5 = 15,$$

$$b_1 = 1, b_2 = 9, b_3 = 36, b_4 = 100, b_5 = 225,$$

$$b_n = \{a_n\}^2,$$

< 6 ページ. 和の記号 Σ (シグマ) 1 >

問の解答

(1) $1 + 2 + 3 + 4 + 5 + 6 + 7$

(2) $1^4 + 2^4 + 3^4 + 4^4$

(3) $3 + 5 + 7 + 9 + 11$

(4) $-7 - 2 + 3 + 8 + 13 + 18$

(5) $1 + 1 + 1 + 1 + 1 + 1 + 1$

< 7 ページ. 和の記号 \sum (シグマ) 2 >

問 1 の解答

$$(1) \sum_{k=1}^n k \quad (2) \sum_{k=1}^n (2k-1) \times 2k \quad (3) \sum_{k=1}^6 (3k-2) \quad (4) \sum_{k=1}^{20} (5k)$$

問 2 の解答

$$(1) 1 + 8 + 17 + 28 + 41$$

$$(2) 10 + 26 + 48 + 76 + 100$$

$$(3) 1 + 4 + 4^2 + \cdots + 4^n$$

< 8 ページ. 和の記号 \sum (シグマ) 3 >

問1の解答

$$(1) 2 \sum_{k=1}^n k + 3 \sum_{k=1}^n 1 = 2 \times \frac{n(n+1)}{2} + 3n = n^2 + 4n$$

$$(2) 8 \sum_{k=1}^n k - 5 \sum_{k=1}^n 1 = 8 \times \frac{n(n+1)}{2} - 5n = 4n^2 - n$$

問2の解答

$$(1) \sum_{k=1}^n (2k - 1) = 2 \sum_{k=1}^n k - \sum_{k=1}^n 1 = 2 \times \frac{n(n+1)}{2} - n = n^2$$

$$(2) \sum_{k=1}^n (5k - 3) = 5 \sum_{k=1}^n k - 3 \sum_{k=1}^n 1 = 5 \times \frac{n(n+1)}{2} - 3n = \frac{5}{2}n^2 - \frac{1}{2}n$$

$$(3) \sum_{k=1}^n (7k - 4) = 7 \sum_{k=1}^n k - 4 \sum_{k=1}^n 1 = 7 \times \frac{n(n+1)}{2} - 4n = \frac{7}{2}n^2 - \frac{1}{2}n$$

< 9 ページ. 和の記号 \sum (シグマ) 4 >

問 1 の解答

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

問 2 の解答

$$(1) \sum_{k=1}^7 k^2 = \frac{7 \times 8 \times (2 \times 7 + 1)}{6} = \frac{7 \times 8 \times 15}{6} = 140$$

$$(2) \sum_{k=1}^{n+1} k^2 = \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

< 10 ページ. 和の記号 \sum (シグマ) 5 >

問 1 の解答

$$\sum_{k=1}^n k^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$$

問 2 の解答

$$(1) \sum_{k=1}^7 k^3 = \left\{ \frac{7 \times 8}{2} \right\}^2 = 28^2 = 784$$

$$(2) \sum_{k=1}^{n-1} k^3 = \left\{ \frac{(n-1)n}{2} \right\}^2$$

< 11 ページ. 和の記号 \sum (シグマ) 6 >

問 1 の解答

(1) $x_2 + x_3 + x_4$

(2) $y_3 + y_4 + y_5 + y_6$

(3) $1^2 + 2^2 + 3^2 + \cdots + n^2$

(4) $2^3 + 3^3 + 4^3 + \cdots + n^3$

問 2 の解答

$$\sum_{i=2}^4 \left\{ \sum_{j=4}^5 (x_i \times y_j) \right\} = \sum_{i=2}^4 2x_i y_i = 2x_2 y_2 + 2x_3 y_3 + 2x_4 y_4$$

< 13 ページ. 区分求積法 2 >**問 1 の解答**

$$\sum_{k=1}^{n-1} k^2 = \boxed{\frac{(n-1)n(2n-1)}{6}}$$

$$S_n = \left\{ \frac{1}{6}(n-1)n(2n-1) \right\} \left(\frac{1}{n} \right)^3 = \frac{1}{6} \left(1 - \boxed{\frac{1}{n}} \right) \left(2 - \boxed{\frac{1}{n}} \right)$$

問 2 の解答

$$S_1 = 0, \quad S_2 = \frac{1}{6} \times \frac{1}{2} \times \frac{3}{2} = \frac{1}{8}, \quad S_3 = \frac{1}{6} \times \frac{2}{3} \times \frac{5}{3} = \frac{5}{27}$$

問 3 の解答

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{1}{6} \times \left(1 - \frac{1}{n} \right) \times \left(2 - \frac{1}{n} \right) = \frac{1}{6} \times 1 \times 2 = \frac{1}{3}$$

< 14 ページ. 区分求積法 3 >

問の解答

$$(1) S_n^* = x_1^2 h + x_2^2 h + x_3^2 h + \cdots + x_n^2 h$$

$$\begin{aligned}(2) S_n^* &= (h)^2 h + (2h)^2 h + (3h)^2 h + \cdots + (nh)^2 h \\ &= (1^2 + 2^2 + 3^2 + \cdots + n^2) h^3 \\ &= \left(\sum_{k=1}^n k^2 \right) \times h^3\end{aligned}$$

$$(3) S_n^* = \left(\frac{n(n+1)(2n+1)}{6} \right) \times \left(\frac{1}{n} \right)^3 = \frac{1}{6} \left(1 + \frac{1}{n} \right) \left(2 + \frac{1}{n} \right)$$

$$(4) S_1^* = \frac{1}{6} \times 2 \times 3 = 1, \quad S_2^* = \frac{1}{6} \times \frac{3}{2} \times \frac{5}{2} = \frac{5}{8}, \quad S_3^* = \frac{1}{6} \times \frac{4}{3} \times \frac{7}{3} = \frac{14}{27}$$

$$(5) \lim_{n \rightarrow \infty} S_n^* = \lim_{n \rightarrow \infty} \frac{1}{6} \times \left(1 + \frac{1}{n} \right) \times \left(2 + \frac{1}{n} \right) = \frac{1}{6} \times 1 \times 2 = \frac{1}{3}$$

$$(6) S = \frac{1}{3}$$

< 15 ページ. 区分求積法 4 >

問 1 の解答

$$\begin{aligned}(1) \quad S_n^* &= h^3 h + (2h^3)h + \cdots + ((n-1)h)^3 h \\ &= \{1^3 + 2^3 + \cdots + (n-1)^3\} h^4 \\ &= \left\{ \sum_{k=1}^{n-1} k^3 \right\} h^4\end{aligned}$$

$$(2) \quad S_n = \left\{ \frac{(n-1)n}{2} \right\}^2 \times \left(\frac{1}{n} \right)^4 = \frac{1}{4} \left(1 - \frac{1}{n} \right)^2$$

$$(3) \quad \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{1}{4} \left(1 - \frac{1}{n} \right)^2 = \frac{1}{4}$$

問 2 の解答

$$S_n^* = \left\{ \sum_{k=1}^{n-1} k^3 \right\} h^4 = \left\{ \frac{(n+1)n}{2} \right\}^2 \times \left(\frac{1}{n} \right)^4 = \frac{1}{4} \left(1 + \frac{1}{n} \right)^2$$

$$\lim_{n \rightarrow \infty} S_n^* = \lim_{n \rightarrow \infty} \frac{1}{4} \left(1 + \frac{1}{n} \right)^2 = \frac{1}{4}$$

問 3 の解答

$$S = \frac{1}{4}$$

< 16 ページ. 区分求積法 5 >

問 1 の解答

$$(1) S_n(x) = x_1^2 h + x_2^2 h + \cdots + x_{n-1}^2 h$$

$$\begin{aligned} (2) S_n(x) &= h^2 h + (2h)^2 h + \cdots + ((n-1)h)^2 h \\ &= \{1^2 + 2^2 + \cdots + (n-1)^2\} h^3 \\ &= \left\{ \sum_{k=1}^{n-1} k^2 \right\} h^3 \end{aligned}$$

$$(3) S_n(x) = \frac{(n-1)n(2n-1)}{6} \times \left(\frac{x}{n}\right)^3 = \frac{1}{6} \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right) x^3$$

$$(4) \lim_{n \rightarrow \infty} S_n(x) = \lim_{n \rightarrow \infty} \frac{1}{6} \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right) x^3 = \frac{1}{3} x^3$$

問 2 の解答

$$\begin{aligned} (1) S_n^*(x) &= x_1^2 h + x_2^2 h + \cdots + x_n^2 h \\ &= \left\{ \sum_{k=1}^n k^2 \right\} h^3 = \frac{n(n+1)(2n+1)}{6} \times \left(\frac{x}{n}\right)^3 = \frac{1}{6} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) x^3 \end{aligned}$$

$$(2) \lim_{n \rightarrow \infty} S_n^*(x) = \lim_{n \rightarrow \infty} \frac{1}{6} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) x^3 = \frac{1}{3} x^3$$

問 3 の解答

$$S(x) = \frac{1}{3} x^3$$

< 17 ページ. 区分求積法 6 >

問 1 の解答

$$(1) S_n(x) = x_1^3 h + x_2^3 h + \cdots + x_{n-1}^3 h$$

$$\begin{aligned}(2) S_n(x) &= h^3 h + (2h^3)h + \cdots + ((n-1)h)^3 h \\ &= (1^3 + 2^3 + \cdots + (n-1)^3)h^4 \\ &= \left\{ \sum_{k=1}^{n-1} k^3 \right\} h^4\end{aligned}$$

$$(3) S_n(x) = \left\{ \frac{(n-1)n}{2} \right\}^2 \left(\frac{x}{n} \right)^4 = \frac{1}{4} \left(1 - \frac{1}{n} \right)^2 x^4$$

$$(4) \lim_{n \rightarrow \infty} S_n(x) = \lim_{n \rightarrow \infty} \frac{1}{4} \left(1 - \frac{1}{n} \right)^2 x^4 = \frac{1}{4} x^4$$

問 2 の解答

$$(1) S_n^*(x) = \left(\sum_{k=1}^n k^3 \right) \left(\frac{x}{n} \right)^4 = \frac{1}{4} \left(1 + \frac{1}{n} \right)^2 x^4$$

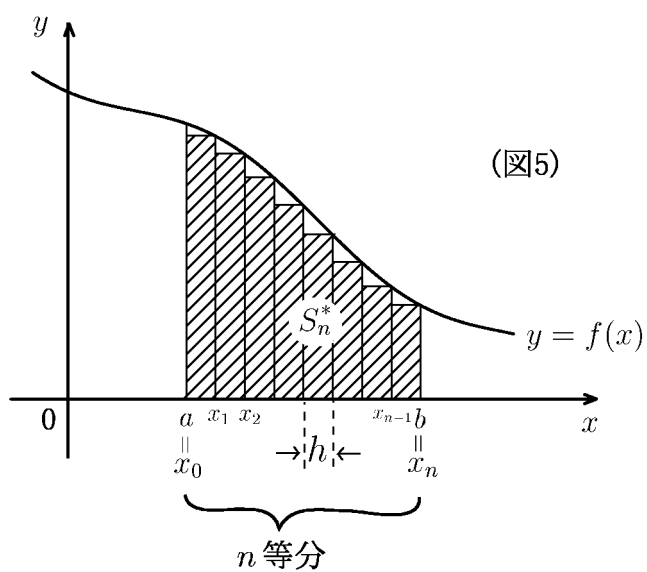
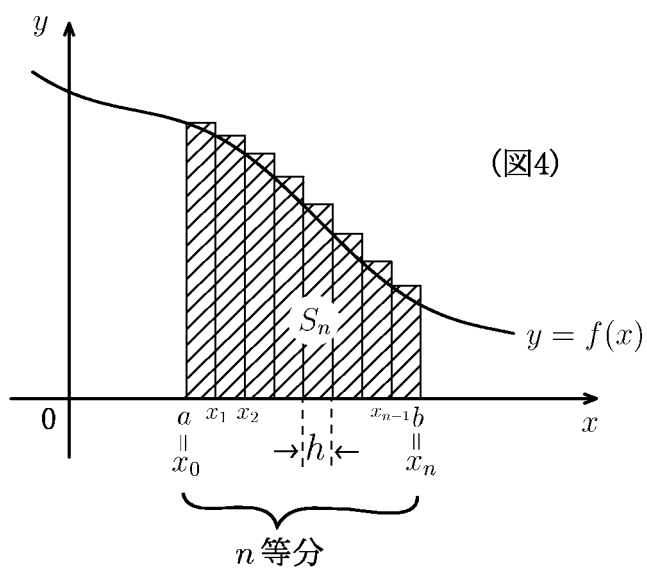
$$(2) \lim_{n \rightarrow \infty} S_n^*(x) = \lim_{n \rightarrow \infty} \frac{1}{4} \left(1 + \frac{1}{n} \right)^2 x^4 = \frac{1}{4} x^4$$

問 3 の解答

$$S(x) = \frac{1}{4} x^4$$

＜ 18 ページ. 定積分の定義 ＞

問の解答



< 21 ページ. 定積分の性質 >

問1の解答

$$\int_0^1 \{-x^3\} dx = - \int_0^1 x^3 dx = -\frac{1}{4}$$

問2の解答

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx = S_1 - S_2$$

< 22 ページ. 面積関数 $S(x)$ >

問1の解答

$$(1) S(x) = x$$

$$(2) S(x) = \frac{1}{2}x^2$$

問2の解答

$$(1) S(x) = \frac{1}{3}x^3$$

$$(2) S(x) = \frac{1}{4}x^4$$

問3の解答

$$(1) S(x) = \frac{1}{5}x^5$$

$$(2) S(x) = \frac{1}{n+1}x^{n+1}$$

問4の解答

$$(S(x))' = f(x) \quad (S(x) \text{ の導関数は } f(x))$$

または

$$\int f(x)dx = S(x) + C$$

< 24 ページ. 微分積分学の基本定理 1 >

問の解答

$$\begin{aligned} \frac{1}{h} \left\{ \int_a^{x+h} f(x) dx - \int_a^x f(x) dx \right\} &= \frac{1}{-\delta} \left\{ \int_a^{x-\delta} f(x) dx - \int_a^x f(x) dx \right\} \\ &= \frac{1}{\delta} \int_{x-\delta}^x f(x) dx = \frac{1}{h} \int_x^{\boxed{x+h}} f(x) dx \end{aligned}$$

< 25 ページ. 微分積分学の基本定理 2 >

問の解答

[定理 8]

< 証明 > 定理 5 と定理 7 より

$$\begin{aligned}
 S'(x) &= \lim_{h \rightarrow 0} \frac{S(x+h) - S(x)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left\{ \int_a^{x+h} f(x) dx - \int_a^x f(x) dx \right\} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \int_{\boxed{x}}^{x+h} f(x) dx = f(x) \quad (\text{証明終})
 \end{aligned}$$

[定理 9]

< 証明 > $S(x) = \int_a^x f(x) dx$ とおくと定理 8 より $S'(x) = f(x)$ だから

$$(F(x) - S(x))' = F'(x) - S'(x) = f(x) - f(x) = 0$$

微分して 0(ゼロ) になる関数 (\Leftrightarrow 傾きが常に 0(ゼロ)) は定数だから

$$F(x) - S(x) = C \quad (C \text{ は定数})$$

とおける。従って

$$F(x) = S(x) + C = \int_a^{\boxed{x}} f(x) dx + C$$

より

$$F(b) - F(a) = \{S(b) + C\} - \{S(a) + C\} = S(b) - S(a)$$

$$= \int_a^{\boxed{b}} f(x) dx - \int_a^{\boxed{a}} f(x) dx = \int_a^b f(x) dx \quad (\text{証明終})$$

< 26 ページ. 定積分 1 >

問の解答

$$(1) [x]_4^7 = 7 - 4 = 3$$

$$(2) \left[\frac{1}{2}x^2 \right]_{-1}^3 = \frac{1}{2} \times 3^2 - \frac{1}{2} \times (-1)^2 = 4$$

$$(3) \left[\frac{1}{3}x^3 \right]_{-2}^1 = \frac{1}{3} \times 1^3 - \frac{1}{3} \times (-2)^3 = \frac{1+8}{3} = 3$$

$$(4) \left[\frac{1}{4}x^4 \right]_{-2}^2 = \frac{1}{4} \times 2^4 - \frac{1}{4}(-2)^4 = 0$$

$$(5) \left[\frac{1}{5}x^5 \right]_{-1}^2 = \frac{1}{5} \times 2^5 - \frac{1}{5}(-1)^5 = \frac{32+1}{5} = \frac{33}{5}$$

< 27 ページ. 定積分 2 >

問 1 の解答

- (1) $\int_a^b dx = \int_a^b 1 dx = [x]_a^b = b - a$
- (2) $\int_a^b x^n dx = \left[\frac{1}{n+1} x^{n+1} \right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$
- (3) $\int_a^b \frac{1}{x} dx = [\log x]_a^b = \log \left(\frac{b}{a} \right)$
- (4) $\int_a^b e^x dx = [e^x]_a^b = e^b - e^a$
- (5) $\int_a^b \cos x dx = [\sin x]_a^b = \sin b - \sin a$
- (6) $\int_a^b \sin x dx = [-\cos x]_a^b = -\cos b + \cos a$

問 2 の解答

- (1) $\int_4^{10} dx = [x]_4^{10} = 10 - 4 = 6$
- (2) $\int_{-1}^1 (x^2 + x^3 + x^4) dx = \left[\frac{1}{3} x^3 + \frac{1}{4} x^4 + \frac{1}{5} x^5 \right]_{-1}^1 = \frac{16}{15}$
- (3) $\int_1^5 \frac{1}{x^2} dx = \left[-\frac{1}{x} \right]_1^5 = \frac{4}{5}$
- (4) $\int_1^2 \frac{1}{x^3} dx = \left[-\frac{1}{2x^2} \right]_1^2 = \frac{3}{8}$
- (5) $\int_4^9 \sqrt{x} dx = \left[\frac{2}{3} x \sqrt{x} \right]_4^9 = \frac{38}{3}$
- (6) $\int_1^8 \sqrt[3]{x} dx = \left[\frac{3}{4} x \sqrt[3]{x} \right]_1^8 = \frac{45}{4}$
- (7) $\int_0^9 \frac{1}{\sqrt{x}} dx = [2\sqrt{x}]_0^9 = 6$
- (8) $\int_1^{e^2} \frac{1}{x} dx = [\log x]_1^{e^2} = 2$
- (9) $\int_2^4 \frac{3}{x} dx = [3 \log x]_2^4 = 3 \log 2$
- (10) $\int_0^2 e^x dx = [e^x]_0^2 = e^2 - 1$
- (11) $\int_{-1}^1 4e^x dx = [4e^x]_{-1}^1 = 4e - \frac{4}{e}$
- (12) $\int_0^\pi \sin x dx = [-\cos x]_0^\pi = 2$
- (13) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$
- (14) $\int_0^{\frac{\pi}{2}} 3 \sin x dx = [-3 \cos x]_0^{\frac{\pi}{2}} = 3$

< 28 ページ. 定積分 3 >

問の解答

$$(1) \quad 2 \int_0^1 x^4 dx = 2 \left[\frac{1}{5} x^5 \right]_0^1 = \frac{2}{5}$$

$$(2) \quad 2 \int_0^1 x^6 dx = 2 \left[\frac{1}{7} x^7 \right]_0^1 = \frac{2}{7}$$

< 29 ページ. 定積分の積分変数 >

問の解答

$$(1) [4t - 4.9t^2]_1^3 = 12 - 4.9 \times 9 - (4 - 4.9) = -31.2$$

$$(2) [\pi r^2]_0^R = \pi R^2$$

$$(3) [-\cos \theta]_0^\pi = 2$$

$$(4) \left[\frac{1}{n+1} u^{n+1} \right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$$

$$(5) \left[\frac{2}{3} u\sqrt{u} \right]_1^9 = \frac{2}{3}(27 - 1) = \frac{52}{3}$$

< 30 ページ. 定積分の置換積分法 1 >

問の解答

$$(1) \quad u = x^3 + 1 \text{ とおくと } \int_{-1}^1 3x^2(x^3 + 1)dx = \int_0^2 u^4 du = \left[\frac{1}{5}u^5 \right]_0^2 = \frac{32}{5}$$

$$(2) \quad u = x^2 + 1 \text{ とおくと } \int_0^2 2x\sqrt{x^2 + 1}dx = \int_1^5 \sqrt{u}du = \left[\frac{2}{3}u\sqrt{u} \right]_1^5 = \frac{2}{3}(5\sqrt{5} - 1)$$

$$(3) \quad u = x^4 + 1 \text{ とおくと } \int_0^1 \frac{4x^3}{(x^4 + 1)^2}dx = \int_1^2 \frac{1}{u^2}du = \left[-\frac{1}{u} \right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

< 31 ページ. 定積分の置換積分法 2 >

問の解答

(1) $u = x^2 + 2$ とおくと $\frac{du}{dx} = 2x$ より、

$$\int_0^1 x(x^2 + 2)^3 dx = \int_2^3 u^3 \times \frac{1}{2} du = \left[\frac{1}{8} u^4 \right]_2^3 = \frac{1}{8}(81 - 16) = \frac{65}{8}$$

(2) $u = x^2$ とおくと、

$$\int_0^3 x e^{x^2} dx = \int_0^9 e^u \times \frac{1}{2} du = \left[\frac{1}{2} e^u \right]_0^9 = \frac{1}{2} e^9 - \frac{1}{2}$$

(3) $u = x^3 + 2$ とおくと、

$$\int_{-1}^2 \frac{x^2}{x^3 + 2} dx = \int_1^{10} \frac{1}{u} \times \frac{1}{3} du = \left[\frac{1}{3} \log u \right]_1^{10} = \frac{1}{3} \log 10$$

(4) $u = x^2 + 1$ とおくと、

$$\int_0^2 \frac{x}{x^2 + 1} dx = \int_1^5 \frac{1}{u^3} \times \frac{1}{2} du = \left[-\frac{1}{4u^2} \right]_1^5 = -\frac{1}{4} \left(\frac{1}{25} - 1 \right) = \frac{6}{25}$$

< 32 ページ. 定積分の部分積分法 >

問の解答

$$\begin{aligned}(1) \quad \int_{-1}^1 (x+1)(x-1)^3 dx &= \int_{-1}^1 (x+1) \times \left(\frac{(x-1)^4}{4} \right)' dx \\ &= \left[(x+1) \times \frac{(x-1)^4}{4} \right]_{-1}^1 - \int_{-1}^1 \frac{(x-1)^4}{4} dx \\ &= \left[-\frac{(x-1)^5}{20} \right]_{-1}^1 = -0 + \frac{(-2)^5}{20} = -\frac{32}{20} = -\frac{8}{5}\end{aligned}$$

$$\begin{aligned}(2) \quad \int_0^{\frac{\pi}{2}} x \sin x dx &= \int_0^{\frac{\pi}{2}} x \times (-\cos x)' dx \\ &= \left[-x \cos x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (-\cos x) dx = \left[\sin x \right]_0^{\frac{\pi}{2}} = 1\end{aligned}$$

$$\begin{aligned}(3) \quad \int_0^1 x e^x dx &= \int_0^1 x (e^x)' dx = \left[x e^x \right]_0^1 - \int_0^1 e^x dx \\ &= e - \left[e^x \right]_0^1 = e - (e - 1) = 1\end{aligned}$$

< 33 ページ. 面積 1 >

問の解答

$$\begin{aligned} S &= \int_{-1}^2 (-x^2 + 2x + 4) dx - \int_{-1}^2 x^2 dx \\ &= \int_{-1}^2 (-2x^2 + 2x + 4) dx \\ &= \left[-\frac{2}{3}x^3 + x^2 + 4x \right]_{-1}^2 \\ &= \left(-\frac{16}{3} + 4 + 8 \right) - \left(\frac{2}{3} + 1 - 4 \right) \\ &= -\frac{18}{3} + 12 + 3 = 9 \end{aligned}$$

< 34 ページ. 面積 2 >**問 1 の解答**

$$\begin{aligned} S &= \int_a^b \{f(x) + C\} dx - \int_a^b \{g(x) + C\} dx \\ &= \int_a^b \{f(x) - g(x)\} dx \end{aligned}$$

問 2 の解答

$$\begin{aligned} S &= \int_{-1}^1 \{(-x^2 + 2x + 1) - (x^2 + 2x - 1)\} dx \\ &= \int_{-1}^1 \{-2x^2 + 2\} dx \\ &= \left[-\frac{2}{3}x^3 + 2x \right]_{-1}^1 \\ &= \left(-\frac{2}{3} + 2 \right) - \left(\frac{2}{3} - 2 \right) = 4 - \frac{4}{3} = \frac{8}{3} \end{aligned}$$

< 35 ページ. 円の面積 >

問の解答

(1) $x = r \sin u$

$$\frac{dx}{du} = r \cos u$$

$$\begin{aligned} \int_0^r \sqrt{r^2 - x^2} dx &= \int_{u=0}^{u=\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 u} r \cos u du \\ &= \int_{u=0}^{u=\frac{\pi}{2}} r^2 \cos^2 u du = \int_0^{\frac{\pi}{2}} \frac{r^2}{2} (1 + \cos(2u)) du \\ &= \left[\frac{r^2}{2} \left(u + \frac{1}{2} \sin(2u) \right) \right]_0^{\frac{\pi}{2}} = \frac{r^2}{2} \left(\frac{\pi}{2} \right) = \frac{\pi}{4} r^2 \end{aligned}$$

(2) (1) より、 $\frac{S}{4} = \frac{\pi}{4} r^2$

 \downarrow

(答) $S = \pi r^2$

< 36 ページ. 体積 1 >

問 1 の解答

$$v_k = f(x_k)h = \frac{25}{98}(x_k)^2h = \frac{25}{98} \times (kh)^2h = \frac{25}{98}k^2 \left(\frac{7}{n}\right)^3 = \frac{175k^2}{2n^3}$$

問 2 の解答

$$V_n = \sum_{k=1}^{n-1} \frac{175}{2n^3}k^2 = \frac{175}{2n^3} \times \frac{(n-1)n(2n-1)}{6} = \frac{175}{12} \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right)$$

問 3 の解答

$$\lim_{n \rightarrow \infty} V_n = \lim_{n \rightarrow \infty} \frac{175}{12} \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right) = \frac{175}{12} \times 1 \times 2 = \frac{175}{6}$$

問 4 の解答

$$v_k = f(x_k)h = \frac{175k^2}{2n^3}$$

問 5 の解答

$$V_n^* = \sum_{k=1}^n \frac{175}{2n^3}k^2 = \frac{175}{2n^3} \times \frac{n(n+1)(2n+1)}{6} = \frac{175}{12} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right)$$

問 6 の解答

$$\lim_{n \rightarrow \infty} V_n^* = \lim_{n \rightarrow \infty} \frac{175}{12} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) = \frac{175}{6}$$

問 7 の解答

$$V = \frac{175}{6}$$

< 37 ページ. 体積 2 >**問 1 の解答**

$$V = \int_0^7 \frac{25}{98} x^2 dx = \left[\frac{25}{98} \times \frac{x^3}{3} \right]_0^7 = \frac{25}{98} \times \frac{7^3}{3} = \frac{175}{6}$$

問 2 の解答

$$(1) f(x) = \frac{1}{2} \times \left(\frac{x}{2}\right) \times \left(\frac{2}{3}x\right) = \frac{x^2}{6}$$

$$(2) V = \int_0^6 f(x) dx = \int_0^6 \frac{x^2}{6} dx = \left[\frac{x^3}{18} \right]_0^6 = \frac{6^3}{18} = 12$$

< 38 ページ. 体積 3 >**問の解答**

$$(1) A_1' C_1' = \frac{5}{7}x$$

$$(2) A_2' C_2' = \frac{5}{7}x$$

$$(3) S(x) = \frac{1}{2} \times \left(\frac{5}{7}x\right)^2 = \frac{25}{98}x^2$$

$$(4) \int_0^7 S(x)dx = \int_0^7 \frac{25}{98}x^2 dx = \left[\frac{25}{98} \times \frac{x^3}{3}\right]_0^7 = \frac{25 \times 7^3}{98 \times 3} = \frac{175}{6}$$

< 39 ページ. 体積 4 >

問の解答

$$S(x) = \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$$

$$V = \int_0^4 S(x)dx = \int_0^4 \frac{x^2}{4}dx = \left[\frac{1}{12}x^3\right] = \frac{4^3}{12} = \frac{16}{3}$$

< 40 ページ. 体積 5 >

問 1 の解答

$$(1) S(x) = \pi \{f(x)\}^2$$

$$(2) V = \int_a^b S(x) dx = \int_a^b \pi \{f(x)\}^2 dx$$

問 2 の解答

$$V = \int_0^h \pi \left(\frac{r}{h}x\right)^2 dx = \int_0^h \frac{\pi r^2}{h^2} x^2 dx = \left[\frac{\pi r^2}{h^2} \times \frac{x^3}{3}\right]_0^h = \frac{\pi r^2}{h^2} \times \frac{h^3}{3} = \frac{\pi r^2 h}{3}$$

問 3 の解答

$$\begin{aligned} V &= \int_{-r}^r \pi \left\{ \sqrt{r^2 - x^2} \right\}^2 dx \\ &= \int_{-r}^r \pi (r^2 - x^2) dx \\ &= \pi \left[r^2 x - \frac{x^3}{3} \right]_{-r}^r \\ &= \pi \left\{ \left(r^3 - \frac{r^3}{3} \right) - \left(-r^3 + \frac{r^3}{3} \right) \right\} \\ &= \frac{4\pi}{3} r^3 \end{aligned}$$