

2002年度 基礎数学ワークブック

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高知工科大学
基礎数学ワークブック
(2002年度版)

Series A

No. 5

解答

< 1 ページ.1対1関数 >

問の解答

(1)1対1である

(2)1対1でない

(3)1対1である

< 2 ページ. 逆関数 1 >

問の解答

$$(1) b = f(a) = 3a - 2$$

$$\downarrow$$

$$3a = b + 2$$

$$a = \frac{b+2}{3} = f^{-1}(b)$$

$$\downarrow$$

$$f^{-1}(b) = \frac{b+2}{3}$$

$$(2) b = f(a) = \frac{1}{a} + 2$$

$$\downarrow$$

$$\frac{1}{a} = b - 2$$

$$a = \frac{1}{b-2} = f^{-1}(b)$$

$$\downarrow$$

$$f^{-1}(b) = \frac{1}{b-2}$$

$$(3) b = f(a) = \sqrt{a}$$

$$\downarrow$$

$$b^2 = a = f^{-1}(b)$$

$$\downarrow$$

$$f^{-1}(b) = b^2$$

< 3 ページ. 逆関数 2 >

問の解答

$$(1) b = f(a) = 3a + 2$$

$$\Downarrow$$

$$3a = b - 2$$

$$\Downarrow$$

$$a = \frac{b-2}{3} = f^{-1}(b)$$

$$\Downarrow$$

$$f^{-1}(x) = \frac{x-2}{3}$$

$$(2) b = f(a) = \frac{1}{a-2}$$

$$\Downarrow$$

$$a-2 = \frac{1}{b}$$

$$\Downarrow$$

$$a = \frac{1}{b} + 2 = f^{-1}(b)$$

$$\Downarrow$$

$$f^{-1}(x) = \frac{1}{x} + 2$$

$$(3) b = f(a) = \sqrt{a}$$

$$\Downarrow$$

$$a = b^2 = f^{-1}(b)$$

$$\Downarrow$$

$$f^{-1}(x) = x^2$$

< 4 ページ. 逆関数 3 >

問の解答

$$b = f(a) = (a + 1)^2$$

$$\Downarrow$$

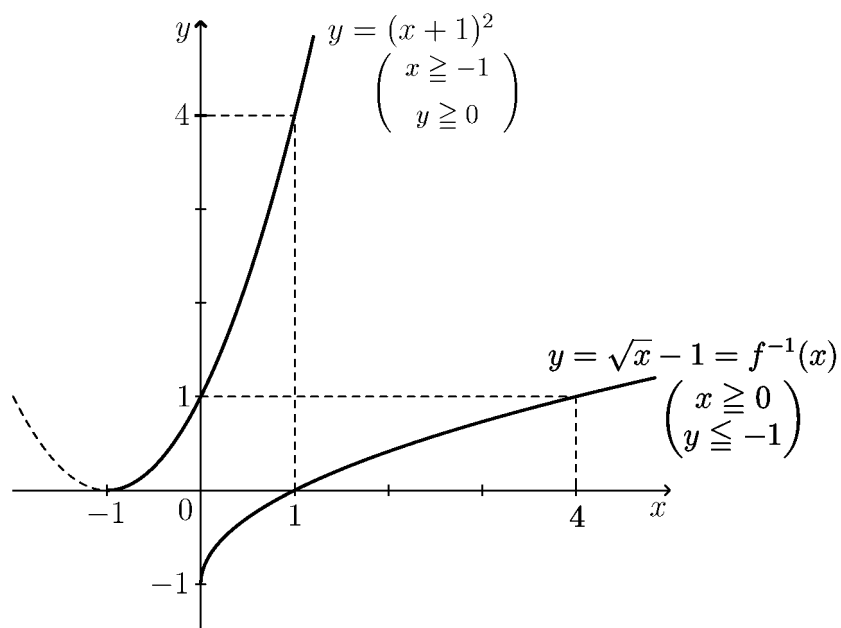
$$a + 1 = \sqrt{b}$$

$$\Downarrow$$

$$a = \sqrt{b} - 1 = f^{-1}(b)$$

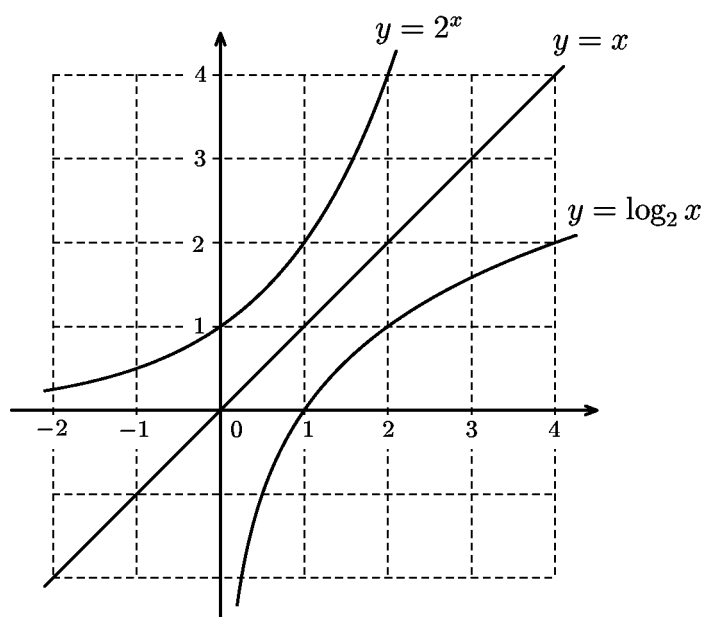
$$\Downarrow$$

$$f^{-1}(x) = \sqrt{x} - 1 \quad (x \geq 0)$$



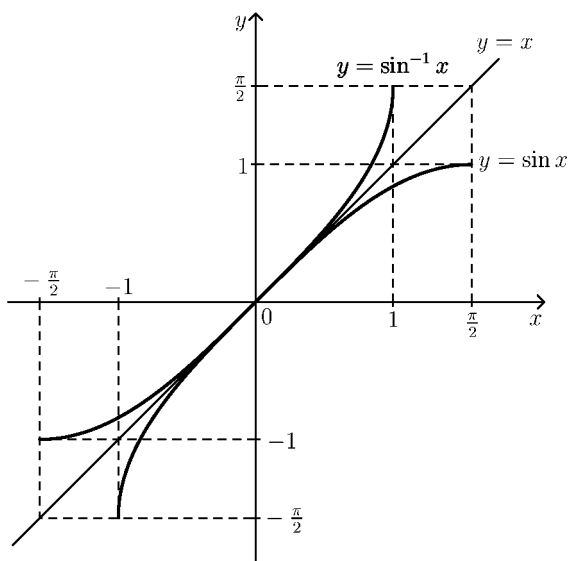
< 5 ページ. 逆関数 4 >

問の解答



< 6 ページ. 逆三角関数 1 >

問1の解答



問2の解答

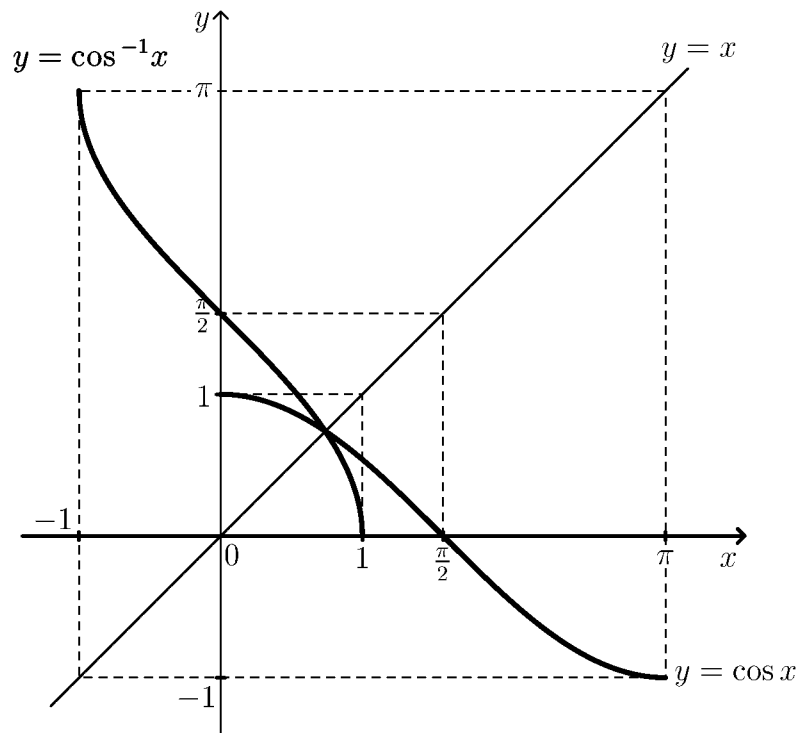
θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

問3の解答

- (1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{3}$ (3) $-\frac{\pi}{6}$

< 7 ページ. 逆三角関数 2 >

問 1 の解答



問 2 の解答

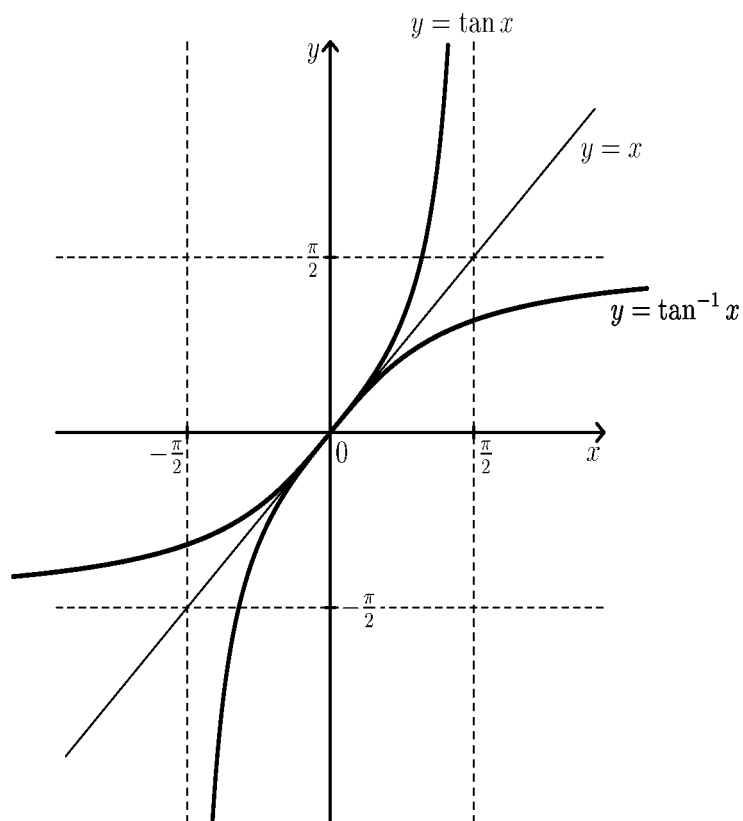
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

問 3 の解答

- (1) $\frac{\pi}{6}$ (2) $\frac{3\pi}{4}$ (3) $\frac{2\pi}{3}$

< 8 ページ. 逆三角関数 3 >

問 1 の解答



問 2 の解答

θ	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\tan \theta$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

問 3 の解答

- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $-\frac{\pi}{3}$

< 9 ページ. 合成関数 1 >

問 1 の解答

$$(1) g(f(x)) = 3x^2 + 3 \quad , \quad f(g(x)) = 9x^2 + 1$$

$$(2) g(f(x)) = (\tan x) + 2 \quad , \quad f(g(x)) = \tan(x + 2)$$

$$(3) g(f(x)) = x - 1 \quad , \quad f(g(x)) = \sqrt{x^2 - 1}$$

$$(4) g(f(x)) = \log_2(x^2 + 2) \quad , \quad f(g(x)) = (\log_2 x)^2 + 2$$

問 2 の解答

$$(1) f(x) = x^2 - x + 2 \quad , \quad g(x) = x^7$$

$$(2) f(x) = 2x + 3 \quad , \quad g(x) = \cos x$$

$$(3) f(x) = 1 - x^2 \quad , \quad g(x) = \sqrt{x}$$

< 10 ページ. 合成関数 2 >

問 1 の解答

$$(1) (f \circ g)(x) = 2(3x - 2) - 1 = 6x - 5, (g \circ f)(x) = 3(2x - 1) - 2 = 6x - 5$$

$$(2) (f \circ g)(x) = (\cos x)^3 = \cos^3 x, (g \circ f)(x) = \cos(x^3)$$

$$(3) (f \circ g)(x) = x^2 + 3x, (g \circ f)(x) = \sqrt{x^4 + 3x^2}$$

$$(4) (f \circ g)(x) = 2^{\log_3 x}, (g \circ f)(x) = \log_3 2^x = x \log_3 2$$

問 2 の解答

$$(1) f^2(x) = f(3x - 2) = 3(3x - 2) - 2 = 9x - 8$$

$$f^3(x) = f(f^2(x)) = f(9x - 8) = 3(9x - 8) - 2 = 27x - 26$$

問 3 の解答

$$(1) f^{-1}(x) = \frac{x+1}{2}$$

$$(2) (f^2 \circ f^{-1})(x) = f^2(f^{-1}(x)) = f^2\left(\frac{x+1}{2}\right) = 4\left(\frac{x+1}{2}\right) - 3 = 2x - 1$$

$$(f^3 \circ f^{-1})(x) = f^3\left(\frac{x+1}{2}\right) = 8\left(\frac{x+1}{2}\right) - 7 = 4x + 4 - 7 = 4x - 3$$

< 11 ページ. 合成関数 3 >

問 1 の解答

$$(1) (f \circ g)(x) = (\sqrt{x})^2 = x \quad , \quad (g \circ f)(x) = \sqrt{x^2} = x$$

$$(2) (f \circ g)(x) = (\sqrt[3]{x})^3 = x \quad , \quad (g \circ f)(x) = \sqrt[3]{x^3} = x$$

問 2 の解答

$$(1) 4$$

$$(2) 16$$

$$(3) 32$$

$$(4) x$$

問 3 の解答

$$(1) 2^{\log_2 x} = x$$

$$(2) \log_2 (2^x) = x$$

問 4 の解答

$$(1) 3$$

$$(2) e^{\log_e 5} = 5$$

$$(3) \frac{\pi}{3}$$

$$(4) 1$$

< 12 ページ. 微分記号 >

問の解答

(1) $2x - 1$

(2) -9.8

(3) $6t - 2$

(4) $2\pi r$

(5) $4\pi r^2$

< 13 ページ. 増分記号 Δ (デルタ) >

問の解答

$$(1) (x^5)' = 5x^4$$

$$(2) (\sin t)' = \cos t$$

$$(3) (\cos u)' = -\sin u$$

< 14 ページ. 合成関数の微分 1 >

問 1 の解答

$$\begin{aligned}\frac{dy}{dx} &= \left(\lim_{\Delta u \rightarrow 0} \frac{\cos(u + \Delta u) - \cos u}{\Delta u} \right) \times \left(\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^4 - x^4}{\Delta x} \right) = (\cos u)' \times (x^4)' \\ &= -\sin(u) \times 4x^3 = -\sin(x^4) \times 4x^3 = -4x^3 \sin(x^4)\end{aligned}$$

問 2 の解答

$$\begin{aligned}\frac{dy}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta u} \times \frac{\Delta u}{\Delta x} \\ &= \left(\lim_{\Delta u \rightarrow 0} \frac{\sin(u + \Delta u) - \sin u}{\Delta u} \right) \times \left(\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 + 2(x + \Delta x)^2 - (x^3 + 2x^2)}{\Delta x} \right) \\ &= (\sin u)' \times (x^3 + 2x^2)' \\ &= \cos u \times (3x^2 + 4x) \\ &= \cos(x^3 + 2x^2) \times (3x^2 + 4x) \\ &= (3x^2 + 4x) \cos(x^3 + 2x^2)\end{aligned}$$

$$\left(\begin{array}{l} \text{ただし} \\ u = x^3 + 2x^2 \\ \Delta u = (x + \Delta x)^3 + 2(x + \Delta x)^2 - (x^3 + 2x^2) \\ \Delta y = \sin((x + \Delta x)^3 + 2(x + \Delta x)^2) - \sin(x^3 + 2x^2) \\ \quad = \sin(u + \Delta u) - \sin u \end{array} \right)$$

< 15 ページ. 合成関数の微分 2 >

問 1 の解答

$$\frac{dy}{du} \times \frac{du}{dx}$$

問 2 の解答

$$(1) \frac{dy}{du} \times \frac{du}{dx} = (u^3)' \times (x^2 - 2x + 5)' = 3u^2(2x - 2) = 6(x - 1)(x^2 - 2x + 5)^2$$

$$(\text{ただし } u = x^2 - 2x + 5)$$

$$(2) \frac{dy}{du} \times \frac{du}{dx} = (\cos u)' \times (2x - 3)' = -\sin(u) \times 2 = -2 \sin(2x - 3)$$

$$(\text{ただし } u = 2x - 3)$$

$$(3) \frac{dy}{du} \times \frac{du}{dx} = (\sin u)' \times (x^5 - 2x^2)' = \cos(u) \times (5x^4 - 4x) = (5x^4 - 4x) \cos(x^5 - 2x^2)$$

$$(\text{ただし } u = x^5 - 2x^2)$$

< 16 ページ. 合成関数の微分 3 >

問 1 の解答

$$(1) (u^7)' \times (3x + 5)' = 7u^6 \times 3 = 21(3x + 5)^6$$

$$(ただし \quad u = 3x + 5)$$

$$(2) (u^8)' \times (4x^2 + 5x)' = 8u^7 \times (8x + 5) = 8(8x + 5)(4x^2 + 5x)^7$$

$$(ただし \quad u = 4x^2 + 5x)$$

問 2 の解答

$$n(f(x))^{n-1} \times f'(x)$$

問 3 の解答

$$(1) 5(3x + 4)^{5-1} \times (3x + 4)'$$

$$= 15(3x + 4)^4$$

$$(2) 6(4x^2 + 9x)^{6-1} \times (4x^2 + 9x)'$$

$$= 6(8x + 9)(4x^2 + 9x)^5$$

$$(3) 10(x^4 - 2x^3)^{10-1} \times (x^4 - 2x^3)'$$

$$= 10(4x^3 - 6x^2)(x^4 - 2x^3)^9$$

$$(4) 5(3 + 4 \sin x)^{5-1} \times (3 + 4 \sin x)'$$

$$= 20 \cos x (3 + 4 \sin x)^4$$

$$(5) 7(x - 3 \cos x)^{5-1} \times (x - 3 \cos x)'$$

$$= 7(1 + 3 \sin x)(x - 3 \cos x)^6$$

< 17 ページ. 合成関数の微分 4 >

問 1 の解答

$$(1) 5 \cos(5x - 4)$$

$$(2) (6x^5 + 14x) \cos(x^6 + 7x^2 - 3)$$

$$(3) -4 \sin(4x + 3)$$

$$(4) -(5x^4 - 2) \sin(x^5 - 2x + 1)$$

問 2 の解答

$$-\sin(f(x)) \times f'(x)$$

問 3 の解答

$$(1) (6x^5 + 35x^4 - 6x + 4) \cos(x^6 + 7x^5 - 3x^2 + 4x)$$

$$(2) (7x^6 - 40x^4 + 12x^2 - 6) \cos(x^7 - 8x^5 + 4x^3 - 6x + 1)$$

< 18 ページ. 合成関数の微分 5 >

問 1 の解答

$$(1) (\log u)' \times (x^3 + 2x - 5)' = \frac{1}{u} \times (3x^2 + 2) = \frac{3x^2 + 2}{x^3 + 2x - 5}$$

$$(\text{ただし } u = x^3 + 2x - 5)$$

$$(2) (\log u)' \times (1 + \sin x)' = \frac{1}{u} \times \cos x = \frac{\cos x}{1 + \sin x}$$

$$(\text{ただし } u = 1 + \sin x)$$

$$(3) (\log u)' \times (5 - \cos x)' = \frac{1}{u} \times (\sin x) = \frac{\sin x}{5 - \cos x}$$

$$(\text{ただし } u = 5 - \cos x)$$

問 2 の解答

$$\frac{f'(x)}{f(x)}$$

問 3 の解答

$$(1) \frac{(x^2 + 2x)'}{x^2 + 2x} = \frac{2x + 2}{x^2 + 2x}$$

$$(2) \frac{(x^6 + 3x^4)'}{x^6 + 3x^4} = \frac{6x^5 + 12x^3}{x^6 + 3x^4} = \frac{6x^2 + 12}{x^3 + 3x}$$

$$(3) \frac{(\sin x)'}{\sin x} = \frac{\cos x}{\sin x}$$

< 19 ページ. 対数微分法 1 >

問 1 の解答

$$\text{(解)} \quad \log y = \log 3^x = x \log 3$$

$$\frac{y'}{y} = \log 3 \Rightarrow y' = y \log 3 = 3^x \log 3$$

$$\text{(答)} \quad (3^x)' = 3^x \log 3$$

問 2 の解答

$$\text{(解)} \quad \log y = \log a^x = x \log a$$

$$\frac{y'}{y} = \log a \Rightarrow y' = y \log a = a^x \log a$$

$$\text{(答)} \quad (a^x)' = a^x \log a$$

問 3 の解答

$$\text{(答)} \quad e^x \log e = e^x$$

< 20 ページ. 対数微分法 2 >

問1の解答

$$\text{(解)} \quad \log y = \log x^{\frac{4}{3}} = \frac{4}{3} \log x$$

$$\frac{y'}{y} = \frac{4}{3} \times \frac{1}{x}$$

$$\Rightarrow y' = y \times \frac{4}{3} \times \frac{1}{x} = x^{\frac{4}{3}} \times \frac{4}{3} \times x^{-1} = \frac{4}{3} \times x^{\frac{4}{3}-1} = \frac{4}{3} \times x^{\frac{1}{3}}$$

$$\text{(答)} \quad (x^{\frac{4}{3}})' = \frac{4}{3} x^{\frac{1}{3}}$$

問2の解答

$$\text{(解)} \quad \log y = \log (x^r) = r \log x$$

$$\frac{y'}{y} = r \times \frac{1}{x} \Rightarrow y' = y \times r \times \frac{1}{x} = x^r \times r \times x^{-1} = r \times x^{r-1}$$

$$\text{(答)} \quad (x^r)' = r x^{r-1}$$

< 21 ページ. x^r の微分 >

問 1 の解答

$$(1) (x^{\frac{5}{4}})' = \frac{5}{4}x^{\frac{1}{4}} = \frac{5}{4}\sqrt[4]{x}$$

$$(2) (x^{\frac{7}{5}})' = \frac{7}{5}x^{\frac{2}{5}} = \frac{7}{5}\sqrt[5]{x^2}$$

$$(3) (x^{\frac{3}{2}})' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$

問 2 の解答

$$(1) -3x^{-4} = -\frac{3}{x^4}$$

$$(2) -4x^{-5} = -\frac{4}{x^5}$$

$$(3) -1 \times x^{-2} = -\frac{1}{x^2}$$

問 3 の解答

$$(1) \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4\sqrt[4]{x^3}}$$

$$(2) \frac{4}{5}x^{-\frac{1}{5}} = \frac{4}{5\sqrt[5]{x}}$$

$$(3) \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

問 4 の解答

$$(1) -\frac{2}{3}x^{-\frac{2}{3}-1} = -\frac{2}{3x\sqrt[3]{x^2}}$$

$$(2) -\frac{1}{4}x^{-\frac{1}{4}-1} = -\frac{1}{4x\sqrt[4]{x}}$$

$$(3) -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2x\sqrt{x}}$$

< 22 ページ. 逆関数の微分 1 >

問 1 の解答

$$y = \cos^{-1} x \Leftrightarrow x = \cos y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\cos y)'} = \frac{1}{-\sin y} = -\frac{1}{\sqrt{1 - \cos^2 y}} = -\frac{1}{\sqrt{1 - x^2}}$$

問 2 の解答

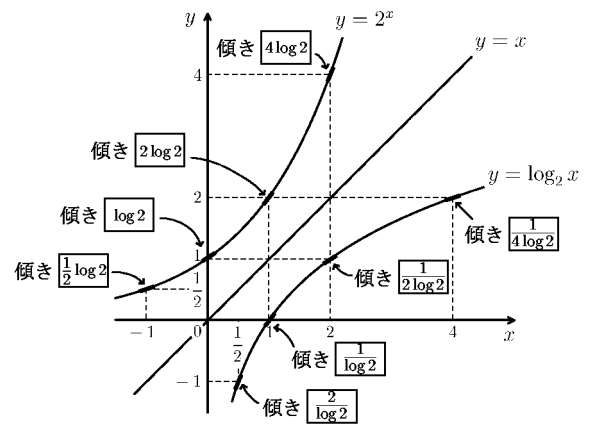
$$y = \tan^{-1} x \Leftrightarrow x = \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\tan y)'} = \frac{1}{\frac{1}{\cos^2 y}} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

< 23 ページ. 逆関数の微分 2 >

問 1 の解答

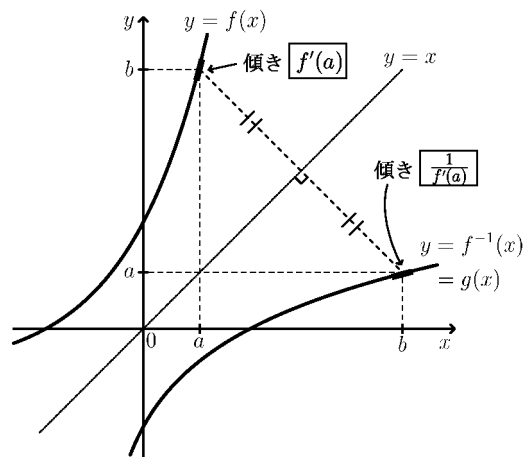
$$\begin{aligned}
 (1) \quad f'(x) &= 2^x \log 2, & g'(x) &= \frac{1}{x} \log_2 e = \frac{1}{x \log 2} \\
 f'(-1) &= \frac{1}{2} \log 2, & g'\left(\frac{1}{2}\right) &= 2 \log_2 e = \frac{2}{\log 2} \\
 f'(0) &= \log 2, & g'(1) &= \log_2 e = \frac{1}{\log 2} \\
 f'(1) &= 2 \log 2, & g'(2) &= \frac{1}{2} \log_2 e = \frac{1}{2 \log 2} \\
 f'(2) &= 4 \log 2, & g'(4) &= \frac{1}{4} \log_2 e = \frac{1}{4 \log 2}
 \end{aligned}$$



$$(2) \quad g'\left(\frac{1}{2}\right) = \frac{1}{f'(-1)}, \quad g'(1) = \frac{1}{f'(0)}, \quad g'(2) = \frac{1}{f'(1)}, \quad g'(4) = \frac{1}{f'(2)}$$

問 2 の解答

$$g'(b) = \frac{1}{f'(a)}$$



問 3 の解答

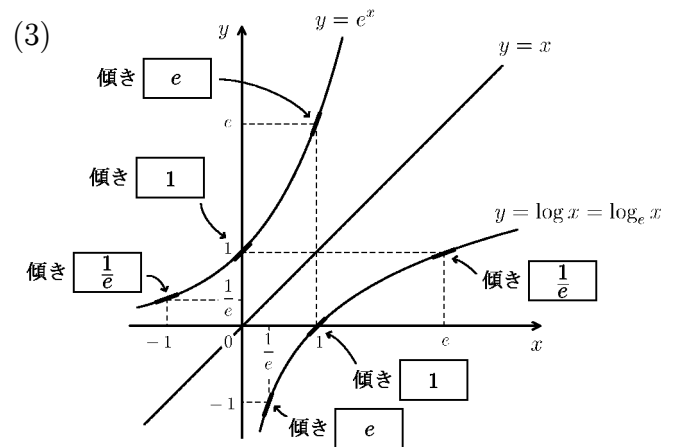
$$(1) \quad f^{-1}(x) = \log x$$

$$(2) \quad f'(x) = e^x, \quad g'(x) = \frac{1}{x}$$

$$f'(-1) = e^{-1} = \frac{1}{e}, \quad g'\left(\frac{1}{e}\right) = \frac{1}{\frac{1}{e}} = e$$

$$f'(0) = e^0 = 1, \quad g'(1) = \frac{1}{1} = 1$$

$$f'(1) = e^1 = e, \quad g'(e) = \frac{1}{e}$$



< 24 ページ. 指数関数の微分 >

問 1 の解答

(1) $3e^{3x}$

(2) $2xe^{x^2+3}$

(3) $(-2x + 2)e^{-x^2+2x}$

問 2 の解答

$$e^{f(x)} \times f'(x)$$

問 3 の解答

(1) $-3e^{-3x}$

(2) $-xe^{-\frac{x^2}{2}}$

< 25 ページ.log|x|の微分 >

問1の解答

$$(1) \frac{dy}{dx} = \frac{(\tan x)'}{\tan x} = \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \frac{1}{\sin x \cos x}$$

$$(2) \frac{dy}{dx} = \frac{2x + 3}{x^2 + 3x}$$

$$(3) \frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

< 26 ページ. 積の微分 1 >

問 1 の解答

$$(\sin x)' \times \log x + \sin x \times (\log x)' = \cos x \log x + \frac{\sin x}{x}$$

問 2 の解答

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - \boxed{f(x)g(x+h)} + \boxed{f(x)g(x+h)} - f(x)g(x)}{h} \\ &= \lim_{h \rightarrow 0} \left\{ \left(\frac{\boxed{f(x+h)} - \boxed{f(x)}}{h} \right) \times g(x+h) + f(x) \times \left(\frac{\boxed{g(x+h)} - \boxed{g(x)}}{h} \right) \right\} \\ &= \left(\boxed{f(x)} \right)' \times g(x) + f(x) \times \left(\boxed{g(x)} \right)' \end{aligned}$$

< 27 ページ. 積の微分 2 >

問の解答

$$(1) (x \sin x)' = \sin x + x \cos x$$

$$(2) (x^2 \cos x)' = 2x \cos x - x^2 \sin x$$

$$(3) (\sin^2 x)' = 2 \sin x \cos x$$

$$(4) (e^x \sin x)' = e^x \sin x + e^x \cos x$$

$$(5) (\sin x \cos x)' = \cos^2 x - \sin^2 x$$

< 28 ページ. 商の微分 1 >

問 1 の解答

$$\left(\frac{1}{\sin x}\right)' = \frac{-(\sin x)'}{\sin^2 x} = -\frac{\cos x}{\sin^2 x}$$

問 2 の解答

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\boxed{g(x)} - \boxed{g(x+h)}}{\boxed{g(x+h)} \times \boxed{g(x)}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\boxed{g(x)} - \boxed{g(x+h)}}{h \times \boxed{g(x+h)} \times \boxed{g(x)}} \\ &= \lim_{h \rightarrow 0} \frac{-\frac{\boxed{g(x+h)} - \boxed{g(x)}}{h}}{\boxed{g(x+h)} \times \boxed{g(x)}} \\ &= \frac{-\left(\boxed{g(x)}\right)'}{\left(\boxed{g(x)}\right)^2} \end{aligned}$$

< 29 ページ. 商の微分 2 >

問の解答

$$(1) \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$(2) \left(\frac{1}{x^2}\right)' = -\frac{2x}{x^4} = -\frac{2}{x^3}$$

$$(3) \left(\frac{1}{x^3}\right)' = -\frac{3x^2}{x^6} = -\frac{3}{x^4}$$

$$(4) \left(\frac{1}{\cos x}\right)' = -\frac{-\sin x}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$$

$$(5) \left(\frac{1}{e^x}\right)' = -\frac{e^x}{(e^x)^2} = -\frac{1}{e^x}$$

< 30 ページ. 分数関数の微分 >

問 1 の解答

$$\begin{aligned}
 \left(\frac{f(x)}{g(x)}\right)' &= \left(f(x) \times \frac{1}{g(x)}\right)' = \left(\boxed{f(x)}\right)' \times \left(\boxed{\frac{1}{g(x)}}\right) + \left(\boxed{f(x)}\right) \times \left(\boxed{\frac{1}{g(x)}}\right)' \\
 &= \frac{\left(\boxed{f(x)}\right)'}{g(x)} + \left(\boxed{f(x)}\right) \times \left(-\frac{\left(\boxed{g(x)}\right)'}{\left(\boxed{g(x)}\right)^2}\right) \\
 &= \frac{\left(\boxed{f(x)}\right)' \times \boxed{g(x)} - \boxed{f(x)} \times \left(\boxed{g(x)}\right)'}{\left(\boxed{g(x)}\right)^2}
 \end{aligned}$$

問 2 の解答

$$(1) \left(\frac{x}{e^x}\right)' = \frac{(x)' \times e^x - x \times (e^x)'}{(e^x)^2} = \frac{e^x - xe^x}{e^{2x}} = \frac{1-x}{e^x}$$

$$\begin{aligned}
 (2) \left(\frac{\sin x}{e^x}\right)' &= \frac{(\sin x)' \times e^x - \sin x \times (e^x)'}{(e^x)^2} = \frac{\cos x \times e^x - \sin x \times e^x}{e^{2x}} \\
 &= \frac{\cos x - \sin x}{e^x}
 \end{aligned}$$

$$(3) \left(\frac{\cos x}{\sin x}\right)' = \frac{(\cos x)' \times \sin x - \cos x \times (\sin x)'}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

< 31 ページ. 微分の練習 >

問1の解答

(1) $(k)' = 0$ (2) $(x^n)' = nx^{n-1}$ (3) $(\sin x)' = \cos x$

(4) $(\cos x)' = -\sin x$ (5) $(\log x)' = \frac{1}{x}$ (6) $(e^x)' = e^x$

(7) $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$ (8) $(\tan^{-1} x)' = \frac{1}{1+x^2}$

問2の解答

(1) $(f(x) + g(x))' = f'(x) + g'(x)$

(2) $(f(x) - g(x))' = f'(x) - g'(x)$

(3) $(kf(x))' = kf'(x)$

(4) $(f(x) \times g(x))' = f'(x)g(x) + f(x)g'(x)$

(5) $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

問3の解答

(1) $((f(x))^n)' = n(f(x))^{n-1} \times f'(x)$

(2) $(\sin(f(x)))' = \cos(f(x)) \times f'(x)$

(3) $(\cos(f(x)))' = -\sin(f(x)) \times f'(x)$

(4) $(\log |f(x)|)' = \frac{f'(x)}{f(x)}$

(5) $(e^{f(x)})' = e^{f(x)} \times f'(x)$

問4の解答

(1) $(x^4 - 5x^3 + 6x^2 - 7x + 8)' = 4x^3 - 15x^2 + 12x - 7$

(2) $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$ (3) $(x\sqrt{x})' = \frac{3}{2}\sqrt{x}$

(4) $\left(\frac{\sin x}{x}\right)' = \frac{x \cos x - \sin x}{x^2}$ (5) $(\sin x \cos x)' = \cos^2 x - \sin^2 x$

(6) $(\tan x)' = \frac{1}{\cos^2 x}$ (7) $(x \log x - x)' = \log x$

(8) $(-\log |\cos x|)' = \tan x$ (9) $(e^{2x} \sin(3x))' = 2e^{2x} \sin(3x) - 3e^{2x} \cos(3x)$

(10) $\left(\log(x + \sqrt{x^2 + 1})\right)' = \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}}$

< 32 ページ. 原始関数 >

問の解答

$$(1) \frac{1}{5}x^5 + C$$

$$(2) \frac{1}{6}x^6 + C$$

$$(3) \frac{1}{7}x^7 + C$$

< 33 ページ. 不定積分 1 >

問 1 の解答

$$(1) \frac{1}{5}x^5 + C$$

$$(2) \frac{1}{6}x^6 + C$$

$$(3) \frac{1}{7}x^7 + C$$

問 2 の解答

$$\frac{1}{n+1}x^{n+1} + C$$

< 34 ページ. 不定積分 2 >

問 1 の解答

$$(1) \frac{3}{4}x^4 + C$$

$$(2) 5x - 2x^2 + C$$

$$(3) x^3 - 5x^2 + 7x + C$$

問 2 の解答

$$\begin{aligned} & \int (4x^2 - 3x + 2 - 4x^2 + 6x + 8) dx \\ &= \int (3x + 10) dx \\ &= \frac{3}{2}x^2 + 10x + C \end{aligned}$$

< 35 ページ. 不定積分 3 >

問 1 の解答

< 微分 >

$$(\sin x)' = \cos x$$

$$(-\cos x)' = \sin x$$

$$(\log |x|)' = \frac{1}{x}$$

$$(e^x)' = e^x$$

< 不定積分 >

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{1}{x} dx = \log |x| + C$$

$$\int e^x dx = e^x + C$$

問 2 の解答

$$(1) \quad -4 \cos x - 5e^x + C$$

$$(2) \quad \frac{1}{2} \log |x| - 3 \sin x + C$$

問 3 の解答

$$(1) \quad \int x^{-4} dx = \frac{1}{-4+1} x^{-4+1} + C = -\frac{1}{3} x^{-3} + C = -\frac{1}{3x^3} + C$$

$$(2) \quad \int x^{\frac{1}{3}} dx = \frac{1}{\frac{1}{3}+1} x^{\frac{1}{3}+1} + C = \frac{3}{4} x^{\frac{4}{3}} + C = \frac{3}{4} x \sqrt[3]{x} + C$$

$$(3) \quad \int x^{-\frac{1}{4}} dx = \frac{1}{-\frac{1}{4}+1} x^{-\frac{1}{4}+1} + C = \frac{4}{3} x^{\frac{3}{4}} + C = \frac{4}{3} \sqrt[4]{x^3} + C$$

< 36 ページ. 合成関数の不定積分 >

問 1 の解答

$$(1) \int e^{f(x)} \times f'(x) dx = e^{f(x)} + C$$

$$(2) \int \cos(f(x)) \times f'(x) dx = \sin(f(x)) + C$$

$$(3) \int \sin(f(x)) \times f'(x) dx = -\cos(f(x)) + C$$

問 2 の解答

$$(1) \log|x^4 + 5x| + C$$

$$(2) \log|\sin x| + C$$

$$(3) e^{x^2+3x} + C$$

$$(4) e^{-\frac{x^2}{2}} + C$$

$$(5) \sin(x^4 + 3) + C$$

$$(6) -\cos\left(\frac{3}{2}x^2 - 2x\right) + C$$

< 37 ページ. 積分記号 >

問の解答

$$(1) 10t - 4.9t^2 + C$$

$$(2) \frac{4\pi r^3}{3} + C$$

$$(3) e^u + C$$

$$(4) \log |y| + C$$

$$(5) \sin u + C$$

< 38 ページ. 置換積分 1 >

問の解答

$$(1) \int \frac{1}{u} \frac{du}{dx} dx = \int \frac{1}{u} du = \log |u| + C = \log |f(x)| + C$$

$$(2) \int \sin(u) \frac{du}{dx} dx = \int \sin(u) du = -\cos(u) + C = -\cos(f(x)) + C$$

$$(3) \int u^n \frac{du}{dx} dx = \int u^n du = \frac{1}{n+1} u^{n+1} + C = \frac{1}{n+1} (f(x))^{n+1} + C$$

< 39 ページ. 置換積分 2 >

問 1 の解答

$$(1) \int \frac{1}{u} \times \frac{1}{a} du = \frac{1}{a} \int \frac{1}{u} du = \frac{1}{a} \log |u| + C = \frac{1}{a} \log |ax + b| + C$$

$$\left(u = ax + b \Rightarrow x = \frac{1}{a} u - \frac{b}{a} \Rightarrow \frac{dx}{du} = \frac{1}{a} \Rightarrow dx = \frac{1}{a} du \right)$$

$$(2) \int \sin(u) \frac{1}{a} du = \frac{1}{a} \int \sin u du = -\frac{1}{a} \cos u + C = -\frac{1}{a} \cos(ax + b) + C$$

$$(3) \int u^n \frac{1}{a} du = \frac{1}{a} \int u^n du = \frac{1}{a} \times \frac{1}{n+1} u^{n+1} + C = \frac{1}{a(n+1)} (ax + b)^{n+1} + C$$

問 2 の解答

$$(1) \frac{1}{4} e^{4x+5} + C$$

$$(2) \frac{1}{3} \sin(3x - 5) + C$$

$$(3) \frac{1}{5} \log |5x + 6| + C$$

$$(4) -\frac{1}{2} \cos(2x + \pi) + C$$

$$(5) \frac{1}{48} (8x + 7)^6 + C$$

$$(6) -\frac{1}{5(5x + 6)} + C$$

< 40 ページ. 置換積分 3 >

問の解答

$$(1) \int x e^u \times \frac{1}{2x} du = \int \frac{1}{2} e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2+1} + C$$

$$\left(u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{1}{2x} du \right)$$

$$(2) \int x^3 e^u \times \frac{1}{4x^3} du = \frac{1}{4} \int e^u du = \frac{1}{4} e^u + C = \frac{1}{4} e^{x^4} + C$$

$$\left(u = x^4 \Rightarrow \frac{du}{dx} = 4x^3 \right)$$

$$(3) \frac{1}{3} \int \cos u du = \frac{1}{3} \sin(x^3 + 2) + C$$

$$(u = x^3 + 2)$$

$$(4) \frac{1}{2} \int \sin u du = -\frac{1}{2} \cos(x^2 + 3) + C$$

$$(u = x^2 + 3)$$

$$(5) \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \log |x^2 + 3| + C$$

$$(u = x^2 + 3)$$

$$(6) \frac{1}{2} \int u^5 du = \frac{1}{2} \times \frac{1}{6} u^6 + C = \frac{1}{12} (x^2 + 1)^6 + C$$

$$(u = x^2 + 1)$$