2002年度 基礎数学ワークブック

高知工科大学

基礎数学ワークブック

(2002年度版)

Series A

No. 5

解答

＜ 1ページ.1対1関数 ＞

問の解答

(1)1 対 1 である

(2)1 対 1 でない

(3)1 対 1 である

＜ 2ページ. 逆関数 1 ＞

問の解答

(1) $b = f(a) = 3a - 2$

$$\Downarrow$$

$$3a = b + 2$$

$$a = \frac{b+2}{3} = f^{-1}(b)$$

$$\Downarrow$$

$$f^{-1}(b) = \frac{b+2}{3}$$

(2) $b = f(a) = \frac{1}{a} + 2$

$$\Downarrow$$

$$\frac{1}{a} = b - 2$$

$$a = \frac{1}{b-2} = f^{-1}(b)$$

$$\Downarrow$$

$$f^{-1}(b) = \frac{1}{b-2}$$

(3) $b = f(a) = \sqrt{a}$

$$\Downarrow$$

$$b^2 = a = f^{-1}(b)$$

$$\Downarrow$$

$$f^{-1}(b) = b^2$$

＜ 3ページ. 逆関数 2 ＞

問の解答

(1) $b = f(a) = 3a + 2$

$\Downarrow$

$3a = b - 2$

$\Downarrow$

$a = \dfrac{b-2}{3} = f^{-1}(b)$

$\Downarrow$

$f^{-1}(x) = \dfrac{x-2}{3}$

(2) $b = f(a) = \dfrac{1}{a-2}$

$\Downarrow$

$a - 2 = \dfrac{1}{b}$

$\Downarrow$

$a = \dfrac{1}{b} + 2 = f^{-1}(b)$

$\Downarrow$

$f^{-1}(x) = \dfrac{1}{x} + 2$

(3) $b = f(a) = \sqrt{a}$

$\Downarrow$

$a = b^2 = f^{-1}(b)$

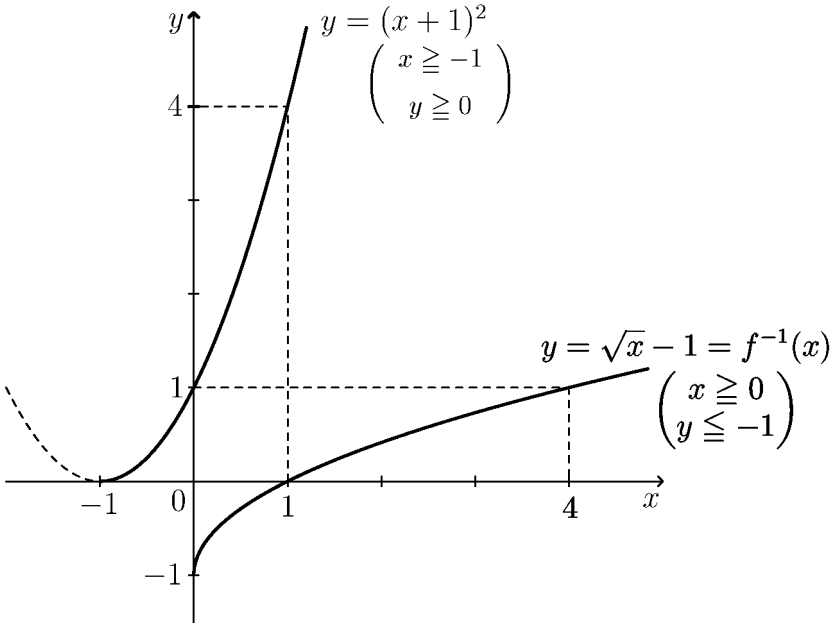
$\Downarrow$

$f^{-1}(x) = x^2$

＜ 4ページ. 逆関数 3 ＞

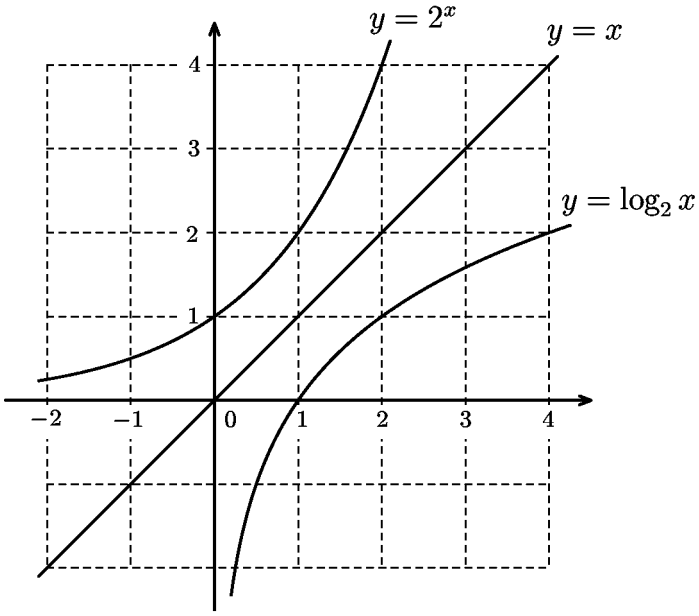
問の解答

$$b = f(a) = (a+1)^2$$
$$\Downarrow$$
$$a + 1 = \sqrt{b}$$
$$\Downarrow$$
$$a = \sqrt{b} - 1 = f^{-1}(b)$$
$$\Downarrow$$
$$f^{-1}(x) = \sqrt{x} - 1 \quad (x \geqq 0)$$



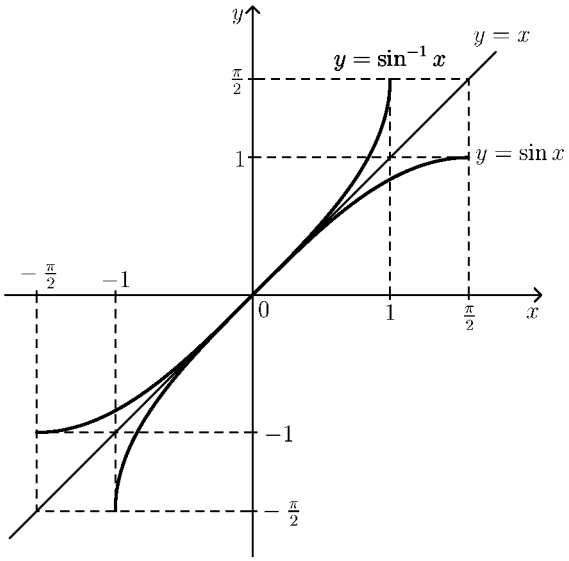
＜ 5ページ. 逆関数 4 ＞

問の解答



＜ 6 ページ. 逆三角関数 1 ＞

問 1 の解答



問 2 の解答

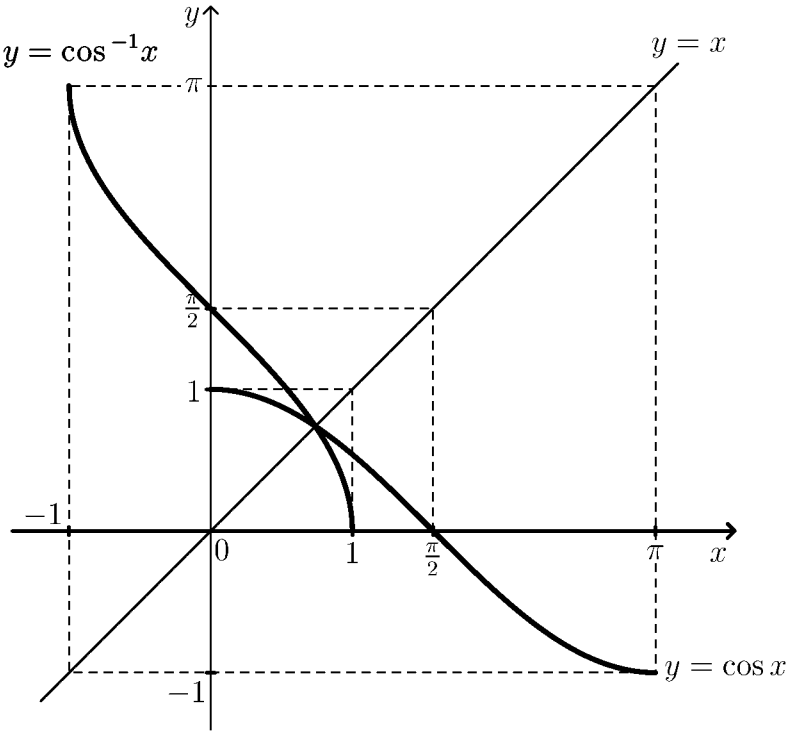
θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

問 3 の解答

(1) $\dfrac{\pi}{4}$　　(2) $-\dfrac{\pi}{3}$　　(3) $-\dfrac{\pi}{6}$

＜ 7ページ. 逆三角関数 2 ＞

問1の解答



問2の解答

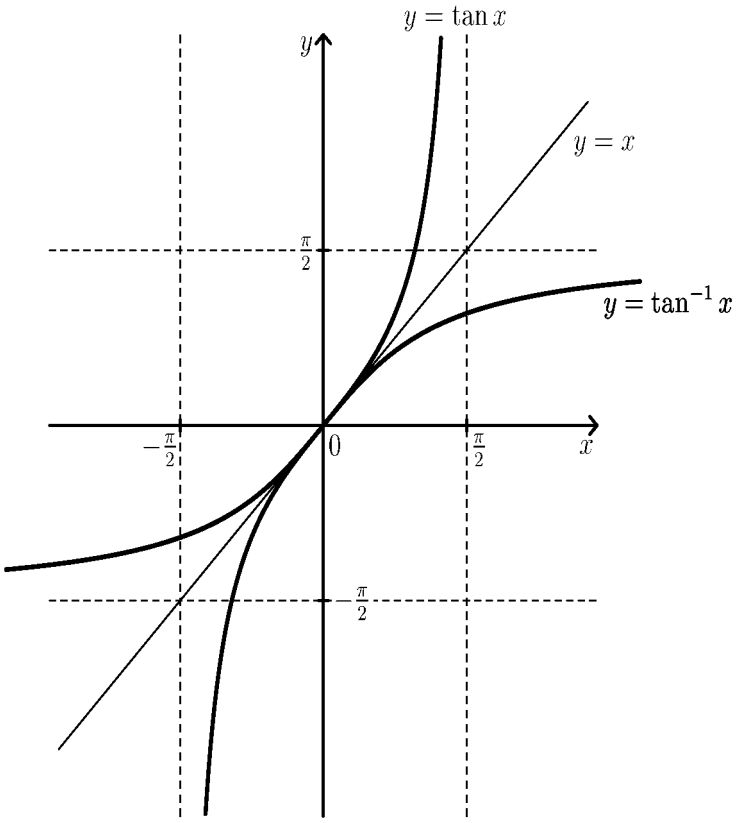
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

問3の解答

(1) $\dfrac{\pi}{6}$　　(2) $\dfrac{3\pi}{4}$　　(3) $\dfrac{2\pi}{3}$

＜ 8 ページ. 逆三角関数 3 ＞

問 1 の解答



問 2 の解答

θ	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\tan\theta$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

問 3 の解答

(1) $\dfrac{\pi}{4}$　　(2) $\dfrac{\pi}{6}$　　(3) $-\dfrac{\pi}{3}$

＜ 9ページ. 合成関数 1 ＞

問1の解答

(1) $g(f(x)) = 3x^2 + 3$, $f(g(x)) = 9x^2 + 1$

(2) $g(f(x)) = (\tan x) + 2$, $f(g(x)) = \tan(x + 2)$

(3) $g(f(x)) = x - 1$, $f(g(x)) = \sqrt{x^2 - 1}$

(4) $g(f(x)) = \log_2(x^2 + 2)$, $f(g(x)) = (\log_2 x)^2 + 2$

問2の解答

(1) $f(x) = x^2 - x + 2$, $g(x) = x^7$

(2) $f(x) = 2x + 3$, $g(x) = \cos x$

(3) $f(x) = 1 - x^2$, $g(x) = \sqrt{x}$

＜ 10ページ. 合成関数 2 ＞

問1の解答

(1) $(f \circ g)(x) = 2(3x-2)-1 = 6x-5$, $(g \circ f)(x) = 3(2x-1)-2 = 6x-5$

(2) $(f \circ g)(x) = (\cos x)^3 = \cos^3 x$, $(g \circ f)(x) = \cos(x^3)$

(3) $(f \circ g)(x) = x^2 + 3x$, $(g \circ f)(x) = \sqrt{x^4+3x^2}$

(4) $(f \circ g)(x) = 2^{\log_3 x}$, $(g \circ f)(x) = \log_3 2^x = x \log_3 2$

問2の解答

(1) $f^2(x) = f(3x-2) = 3(3x-2)-2 = 9x-8$

$f^3(x) = f(f^2(x) = f(9x-8) = 3(9x-8)-2 = 27x-26$

問3の解答

(1) $f^{-1}(x) = \dfrac{x+1}{2}$

(2) $(f^2 \circ f^{-1})(x) = f^2(f^{-1}(x)) = f^2\left(\dfrac{x+1}{2}\right) = 4\left(\dfrac{x+1}{2}\right) - 3 = 2x-1$

$(f^3 \circ f^{-1}) = f^3\left(\dfrac{x+1}{2}\right) = 8\left(\dfrac{x+1}{2}\right) - 7 = 4x+4-7 = 4x-3$

＜ 11 ページ. 合成関数 3 ＞

問 1 の解答

(1) $(f \circ g)(x) = (\sqrt{x})^2 = x$, $(g \circ f)(x) = \sqrt{x^2} = x$

(2) $(f \circ g)(x) = (\sqrt[3]{x})^3 = x$, $(g \circ f)(x) = \sqrt[3]{x^3} = x$

問 2 の解答

(1) 4

(2) 16

(3) 32

(4) x

問 3 の解答

(1) $2^{\log_2 x} = x$

(2) $\log_2 (2^x) = x$

問 4 の解答

(1) 3

(2) $e^{\log_e 5} = 5$

(3) $\dfrac{\pi}{3}$

(4) 1

＜ 12ページ. 微分記号 ＞

問の解答

(1) $2x - 1$

(2) -9.8

(3) $6t - 2$

(4) $2\pi r$

(5) $4\pi r^2$

< 13ページ. 増分記号 Δ(デルタ) >

問の解答

(1) $(x^5)' = 5x^4$

(2) $(\sin t)' = \cos t$

(3) $(\cos u)' = -\sin u$

＜ 14ページ. 合成関数の微分 1 ＞

問1の解答

$$\frac{dy}{dx} = \left(\lim_{\Delta u \to 0} \frac{\cos(u+\Delta u) - \cos u}{\Delta u}\right) \times \left(\lim_{\Delta x \to 0} \frac{(x+\Delta x)^4 - x^4}{\Delta x}\right) = (\cos u)' \times (x^4)'$$

$$= -\sin(u) \times 4x^3 = -\sin(x^4) \times 4x^3 = -4x^3 \sin(x^4)$$

問2の解答

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \times \frac{\Delta u}{\Delta x}$$

$$= \left(\lim_{\Delta u \to 0} \frac{\sin(u+\Delta u) - \sin u}{\Delta u}\right) \times \left(\lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 + 2(x+\Delta x)^2 - (x^3+2x^2)}{\Delta x}\right)$$

$$= (\sin u)' \times (x^3+2x^2)'$$

$$= \cos u \times (3x^2+4x)$$

$$= \cos(x^3+2x^2) \times (3x^2+4x)$$

$$= (3x^2+4x)\cos(x^3+2x^2)$$

$$\left(\begin{array}{l} \text{ただし} \\ \quad u = x^3+2x^2 \\ \Delta u = (x+\Delta x)^3 + 2(x+\Delta x)^2 - (x^3+x^2) \\ \Delta y = \sin((x+\Delta x)^3 + 2(x+\Delta x)^2) - \sin(x^3+2x^2) \\ \quad\quad = \sin(u+\Delta u) - \sin u \end{array}\right)$$

＜ 15 ページ. 合成関数の微分 2 ＞

問 1 の解答

$$\frac{dy}{du} \times \frac{du}{dx}$$

問 2 の解答

(1) $\dfrac{dy}{du} \times \dfrac{du}{dx} = (u^3)' \times (x^2 - 2x + 5)' = 3u^2(2x - 2) = 6(x - 1)(x^2 - 2x + 5)^2$

(ただし　$u = x^2 - 2x + 5$)

(2) $\dfrac{dy}{du} \times \dfrac{du}{dx} = (\cos u)' \times (2x - 3)' = -\sin(u) \times 2 = -2\sin(2x - 3)$

(ただし　$u = 2x - 3$)

(3) $\dfrac{dy}{du} \times \dfrac{du}{dx} = (\sin u)' \times (x^5 - 2x^2)' = \cos(u) \times (5x^4 - 4x) = (5x^4 - 4x)\cos(x^5 - 2x^2)$

(ただし　$u = x^5 - 2x^2$)

＜ 16 ページ. 合成関数の微分 3 ＞

問 1 の解答

(1) $(u^7)' \times (3x+5)' = 7u^6 \times 3 = 21(3x+5)^6$

(ただし　$u = 3x+5$)

(2) $(u^8)' \times (4x^2+5x)' = 8u^7 \times (8x+5) = 8(8x+5)(4x^2+5x)^7$

(ただし　$u = 4x^2+5x$)

問 2 の解答

$n(f(x))^{n-1} \times f'(x)$

問 3 の解答

(1) $5(3x+4)^{5-1} \times (3x+4)'$

$= 15(3x+4)^4$

(2) $6(4x^2+9x)^{6-1} \times (4x^2+9x)'$

$= 6(8x+9)(4x^2+9x)^5$

(3) $10(x^4-2x^3)^{10-1} \times (x^4-2x^3)'$

$= 10(4x^3-6x^2)(x^4-2x^3)^9$

(4) $5(3+4\sin x)^{5-1} \times (3+4\sin x)'$

$= 20\cos x(3+4\sin x)^4$

(5) $7(x-3\cos x)^{5-1} \times (x-3\cos x)'$

$= 7(1+3\sin x)(x-3\cos x)^6$

＜ 17ページ. 合成関数の微分 4 ＞

問1の解答

(1) $5\cos(5x-4)$

(2) $(6x^5+14x)\cos(x^6+7x^2-3)$

(3) $-4\sin(4x+3)$

(4) $-(5x^4-2)\sin(x^5-2x+1)$

問2の解答

$-\sin(f(x))\times f'(x)$

問3の解答

(1) $(6x^5+35x^4-6x+4)\cos(x^6+7x^5-3x^2+4x)$

(2) $(7x^6-40x^4+12x^2-6)\cos(x^7-8x^5+4x^3-6x+1)$

＜ 18 ページ. 合成関数の微分 5 ＞

問 1 の解答

(1) $(\log u)' \times (x^3+2x-5)' = \dfrac{1}{u} \times (3x^2+2) = \dfrac{3x^2+2}{x^3+2x-5}$

(ただし　$u = x^3+2x-5$)

(2) $(\log u)' \times (1+\sin x)' = \dfrac{1}{u} \times \cos x = \dfrac{\cos x}{1+\sin x}$

(ただし　$u = 1+\sin x$)

(3) $(\log u)' \times (5-\cos x)' = \dfrac{1}{u} \times (\sin x) = \dfrac{\sin x}{5-\cos x}$

(ただし　$u = 5-\cos x$)

問 2 の解答

$$\frac{f'(x)}{f(x)}$$

問 3 の解答

(1) $\dfrac{(x^2+2x)'}{x^2+2x} = \dfrac{2x+2}{x^2+2x}$

(2) $\dfrac{(x^6+3x^4)'}{x^6+3x^4} = \dfrac{6x^5+12x^3}{x^6+3x^4} = \dfrac{6x^2+12}{x^3+3x}$

(3) $\dfrac{(\sin x)'}{\sin x} = \dfrac{\cos x}{\sin x}$

＜ 19 ページ. 対数微分法 1 ＞

問 1 の解答

(解) $\log y = \log 3^x = x \log 3$

$\frac{y'}{y} = \log 3 \Rightarrow y' = y \log 3 = 3^x \log 3$

(答) $(3^x)' = 3^x \log 3$

問 2 の解答

(解) $\log y = \log a^x = x \log a$

$\frac{y'}{y} = \log a \Rightarrow y' = y \log a = a^x \log a$

(答) $(a^x)' = a^x \log a$

問 3 の解答

(答) $e^x \log e = e^x$

＜ 20 ページ. 対数微分法 2 ＞

問 1 の解答

(解) $\log y = \log x^{\frac{4}{3}} = \dfrac{4}{3}\log x$

$$\frac{y'}{y} = \frac{4}{3} \times \frac{1}{x}$$
$$\Rightarrow y' = y \times \frac{4}{3} \times \frac{1}{x} = x^{\frac{4}{3}} \times \frac{4}{3} \times x^{-1} = \frac{4}{3} \times x^{\frac{4}{3}-1} = \frac{4}{3} \times x^{\frac{1}{3}}$$

(答) $(x^{\frac{4}{3}})' = \dfrac{4}{3}x^{\frac{1}{3}}$

問 2 の解答

(解) $\log y = \log(x^r) = r\log x$

$$\frac{y'}{y} = r \times \frac{1}{x} \Rightarrow y' = y \times r \times \frac{1}{x} = x^r \times r \times x^{-1} = r \times x^{r-1}$$

(答) $(x^r)' = rx^{r-1}$

＜ 21 ページ.x^r の微分 ＞

問 1 の解答

(1) $(x^{\frac{5}{4}})' = \dfrac{5}{4}x^{\frac{1}{4}} = \dfrac{5}{4}\sqrt[4]{x}$

(2) $(x^{\frac{7}{5}})' = \dfrac{7}{5}x^{\frac{2}{5}} = \dfrac{7}{5}\sqrt[5]{x^2}$

(3) $(x^{\frac{3}{2}})' = \dfrac{3}{2}x^{\frac{1}{2}} = \dfrac{3}{2}\sqrt{x}$

問 2 の解答

(1) $-3x^{-4} = -\dfrac{3}{x^4}$

(2) $-4x^{-5} = -\dfrac{4}{x^5}$

(3) $-1 \times x^{-2} = -\dfrac{1}{x^2}$

問 3 の解答

(1) $\dfrac{1}{4}x^{-\frac{3}{4}} = \dfrac{1}{4\sqrt[4]{x^3}}$

(2) $\dfrac{4}{5}x^{-\frac{1}{5}} = \dfrac{4}{5\sqrt[5]{x}}$

(3) $\dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

問 4 の解答

(1) $-\dfrac{2}{3}x^{-\frac{2}{3}-1} = -\dfrac{2}{3x\sqrt[3]{x^2}}$

(2) $-\dfrac{1}{4}x^{-\frac{1}{4}-1} = -\dfrac{1}{4x\sqrt[4]{x}}$

(3) $-\dfrac{1}{2}x^{-\frac{1}{2}-1} = -\dfrac{1}{2x\sqrt{x}}$

＜ 22 ページ. 逆関数の微分 1 ＞

問 1 の解答

$$y = \cos^{-1} x \Leftrightarrow x = \cos y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\cos y)'} = \frac{1}{-\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$$

問 2 の解答

$$y = \tan^{-1} x \Leftrightarrow x = \tan y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{(\tan y)'} = \frac{1}{\frac{1}{\cos^2 y}} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

＜ 23 ページ. 逆関数の微分 2 ＞

問 1 の解答

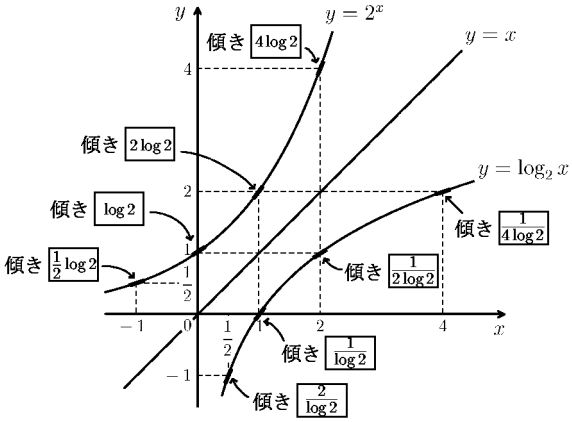
(1) $f'(x) = 2^x \log 2$, $g'(x) = \dfrac{1}{x}\log_2 e = \dfrac{1}{x\log 2}$

$f'(-1) = \dfrac{1}{2}\log 2$, $g'\left(\dfrac{1}{2}\right) = 2\log_2 e = \dfrac{2}{\log 2}$

$f'(0) = \log 2$, $g'(1) = \log_2 e = \dfrac{1}{\log 2}$

$f'(1) = 2\log 2$, $g'(2) = \dfrac{1}{2}\log_2 e = \dfrac{1}{2\log 2}$

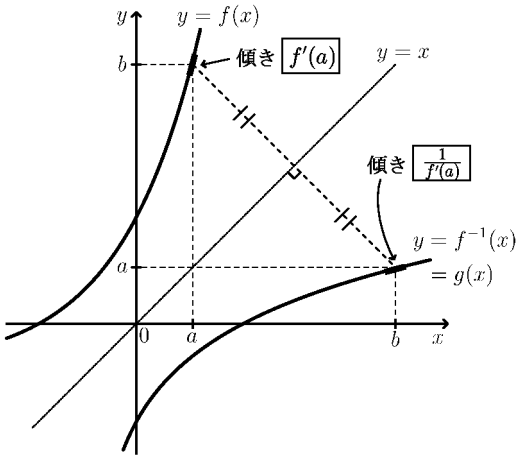
$f'(2) = 4\log 2$, $g'(4) = \dfrac{1}{4}\log_2 e = \dfrac{1}{4\log 2}$



(2) $g'\left(\dfrac{1}{2}\right) = \dfrac{1}{f'(-1)}$, $g'(1) = \dfrac{1}{f'(0)}$, $g'(2) = \dfrac{1}{f'(1)}$, $g'(4) = \dfrac{1}{f'(2)}$

問 2 の解答

$g'(b) = \dfrac{1}{f'(a)}$



問 3 の解答

(1) $f^{-1}(x) = \log x$

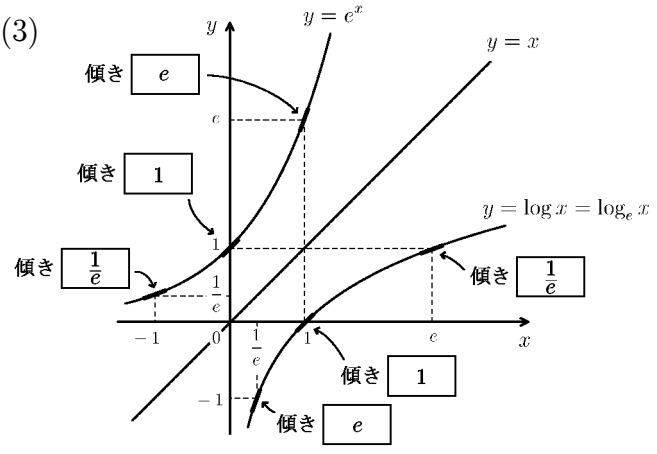
(2) $f'(x) = e^x$, $g'(x) = \dfrac{1}{x}$

$f'(-1) = e^{-1} = \dfrac{1}{e}$, $g'\left(\dfrac{1}{e}\right) = \dfrac{1}{\frac{1}{e}} = e$

$f'(0) = e^0 = 1$, $g'(1) = \dfrac{1}{1} = 1$

$f'(1) = e^1 = e$, $g'(e) = \dfrac{1}{e}$

(3)



＜ 24 ページ. 指数関数の微分 ＞

問 1 の解答

(1) $3e^{3x}$

(2) $2xe^{x^2+3}$

(3) $(-2x+2)e^{-x^2+2x}$

問 2 の解答

$e^{f(x)} \times f'(x)$

問 3 の解答

(1) $-3e^{-3x}$

(2) $-xe^{-\frac{x^2}{2}}$

$<$ 25 ページ.$\log|x|$ の微分 $>$

問 1 の解答

(1) $\dfrac{dy}{dx} = \dfrac{(\tan x)'}{\tan x} = \dfrac{\dfrac{1}{\cos^2 x}}{\dfrac{\sin x}{\cos x}} = \dfrac{1}{\sin x \cos x}$

(2) $\dfrac{dy}{dx} = \dfrac{2x+3}{x^2+3x}$

(3) $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

＜ 26 ページ. 積の微分 1 ＞

問 1 の解答

$$(\sin x)' \times \log x + \sin x \times (\log x)' = \cos x \log x + \frac{\sin x}{x}$$

問 2 の解答

$$\lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - \boxed{f(x)g(x+h)} + \boxed{f(x)g(x+h)} - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \left\{ \left(\frac{\boxed{f(x+h)} - \boxed{f(x)}}{h} \right) \times g(x+h) + f(x) \times \left(\frac{\boxed{g(x+h)} - \boxed{g(x)}}{h} \right) \right\}$$

$$= \left(\boxed{f(x)} \right)' \times g(x) + f(x) \times \left(\boxed{g(x)} \right)'$$

＜ 27 ページ. 積の微分 2 ＞

問の解答

(1) $(x\sin x)' = \sin x + x\cos x$

(2) $(x^2\cos x)' = 2x\cos x - x^2\sin x$

(3) $(\sin^2 x)' = 2\sin x\cos x$

(4) $(e^x\sin x)' = e^x\sin x + e^x\cos x$

(5) $(\sin x\cos x)' = \cos^2 x - \sin^2 x$

＜ 28 ページ. 商の微分 1 ＞

問 1 の解答

$$\left(\frac{1}{\sin x}\right)' = \frac{-(\sin x)'}{\sin^2 x} = -\frac{\cos x}{\sin^2 x}$$

問 2 の解答

$$\lim_{h\to 0} \frac{\dfrac{1}{g(x+h)} - \dfrac{1}{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{\boxed{g(x)} - \boxed{g(x+h)}}{\boxed{g(x+h)} \times \boxed{g(x)}}}{h}$$

$$= \lim_{h\to 0} \frac{\boxed{g(x)} - \boxed{g(x+h)}}{h \times \boxed{g(x+h)} \times \boxed{g(x)}}$$

$$= \lim_{h\to 0} \frac{-\dfrac{\boxed{g(x+h)} - \boxed{g(x)}}{h}}{\boxed{g(x+h)} \times \boxed{g(x)}}$$

$$= \frac{-\left(\boxed{g(x)}\right)'}{\left(\boxed{g(x)}\right)^2}$$

＜ 29 ページ. 商の微分 2 ＞

問の解答

(1) $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$

(2) $\left(\dfrac{1}{x^2}\right)' = -\dfrac{2x}{x^4} = -\dfrac{2}{x^3}$

(3) $\left(\dfrac{1}{x^3}\right)' = -\dfrac{3x^2}{x^6} = -\dfrac{3}{x^4}$

(4) $\left(\dfrac{1}{\cos x}\right)' = -\dfrac{-\sin x}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x}$

(5) $\left(\dfrac{1}{e^x}\right)' = -\dfrac{e^x}{(e^x)^2} = -\dfrac{1}{e^x}$

＜ 30 ページ. 分数関数の微分 ＞

問 1 の解答

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x)\times\frac{1}{g(x)}\right)' = \left(\boxed{f(x)}\right)'\times\left(\frac{1}{\boxed{g(x)}}\right) + \left(\boxed{f(x)}\right)\times\left(\frac{1}{\boxed{g(x)}}\right)'$$

$$= \frac{\left(\boxed{f(x)}\right)'}{g(x)} + \left(\boxed{f(x)}\right)\times\left(-\frac{\left(\boxed{g(x)}\right)'}{\left(\boxed{g(x)}\right)^2}\right)$$

$$= \frac{\left(\boxed{f(x)}\right)'\times\boxed{g(x)} - \boxed{f(x)}\times\left(\boxed{g(x)}\right)'}{\left(g(x)\right)^2}$$

問 2 の解答

(1) $\left(\dfrac{x}{e^x}\right)' = \dfrac{(x)'\times e^x - x\times(e^x)'}{(e^x)^2} = \dfrac{e^x - xe^x}{e^{2x}} = \dfrac{1-x}{e^x}$

(2) $\left(\dfrac{\sin x}{e^x}\right)' = \dfrac{(\sin x)'\times e^x - \sin x\times(e^x)'}{(e^x)^2} = \dfrac{\cos x\times e^x - \sin x\times e^x}{e^{2x}}$

$= \dfrac{\cos x - \sin x}{e^x}$

(3) $\left(\dfrac{\cos x}{\sin x}\right)' = \dfrac{(\cos x)'\times\sin x - \cos x\times(\sin x)'}{\sin^2 x} = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\dfrac{1}{\sin^2 x}$

＜ 31 ページ. 微分の練習 ＞

問 1 の解答

(1) $(k)' = 0$ (2) $(x^n)' = nx^{n-1}$ (3) $(\sin x)' = \cos x$

(4) $(\cos x)' = -\sin x$ (5) $(\log x)' = \dfrac{1}{x}$ (6) $(e^x)' = e^x$

(7) $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$ (8) $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$

問 2 の解答

(1) $\Big(f(x) + g(x)\Big)' = f'(x) + g'(x)$

(2) $\Big(f(x) - g(x)\Big)' = f'(x) - g'(x)$

(3) $\Big(kf(x)\Big)' = kf'(x)$

(4) $\Big(f(x) \times g(x)\Big)' = f'(x)g(x) + f(x)g'(x)$

(5) $\Big(\dfrac{f(x)}{g(x)}\Big)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{\Big(g(x)\Big)^2}$

問 3 の解答

(1) $\Big(\big(f(x)\big)^n\Big)' = n\Big(f(x)\Big)^{n-1} \times f'(x)$

(2) $\Big(\sin\big(f(x)\big)\Big)' = \cos\big(f(x)\big) \times f'(x)$

(3) $\Big(\cos\big(f(x)\big)\Big)' = -\sin\big(f(x)\big) \times f'(x)$

(4) $\Big(\log|f(x)|\Big)' = \dfrac{f'(x)}{f(x)}$

(5) $\Big(e^{f(x)}\Big)' = e^{f(x)} \times f'(x)$

問 4 の解答

(1) $(x^4 - 5x^3 + 6x^2 - 7x + 8)' = 4x^3 - 15x^2 + 12x - 7$

(2) $(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$ (3) $(x\sqrt{x})' = \dfrac{3}{2}\sqrt{x}$

(4) $\Big(\dfrac{\sin x}{x}\Big)' = \dfrac{x\cos x - \sin x}{x^2}$ (5) $(\sin x \cos x)' = \cos^2 x - \sin^2 x$

(6) $(\tan x)' = \dfrac{1}{\cos^2 x}$ (7) $(x\log x - x)' = \log x$

(8) $(-\log|\cos x|)' = \tan x$ (9) $\Big(e^{2x}\sin(3x)\Big)' = 2e^{2x}\sin(3x) - 3e^{2x}\cos(3x)$

(10) $\Big(\log\big(x + \sqrt{x^2+1}\big)\Big)' = \dfrac{1 + \frac{x}{\sqrt{x^2+1}}}{x + \sqrt{x^2+1}} = \dfrac{1}{\sqrt{x^2+1}}$

＜ 32 ページ. 原始関数 ＞

問の解答

(1) $\dfrac{1}{5}x^5 + C$

(2) $\dfrac{1}{6}x^6 + C$

(3) $\dfrac{1}{7}x^7 + C$

＜ 33 ページ. 不定積分 1 ＞

問 1 の解答

(1) $\dfrac{1}{5}x^5 + C$

(2) $\dfrac{1}{6}x^6 + C$

(3) $\dfrac{1}{7}x^7 + C$

問 2 の解答

$\dfrac{1}{n+1}x^{n+1} + C$

＜ 34 ページ. 不定積分 2 ＞

問 1 の解答

(1) $\dfrac{3}{4}x^4 + C$

(2) $5x - 2x^2 + C$

(3) $x^3 - 5x^2 + 7x + C$

問 2 の解答

$$\int \left(4x^2 - 3x + 2 - 4x^2 + 6x + 8\right) dx$$

$$= \int (3x + 10)\, dx$$

$$= \frac{3}{2}x^2 + 10x + C$$

＜ 35 ページ. 不定積分 3 ＞

問 1 の解答

＜ 微分 ＞	＜ 不定積分 ＞
$(\sin x)' = \cos x$	$\int \cos x dx = \sin x \ + \ C$
$(-\cos x)' = \sin x$	$\int \sin x dx = -\cos x \ + \ C$
$(\log \lvert x \rvert)' = \ \dfrac{1}{x}$	$\int \dfrac{1}{x} dx = \log \lvert x \rvert \ + \ C$
$(e^x)' = e^x$	$\int e^x dx = e^x \ + \ C$

問 2 の解答

(1) $-4\cos x - 5e^x + C$

(2) $\dfrac{1}{2}\log|x| - 3\sin x + C$

問 3 の解答

(1) $\displaystyle\int x^{-4} dx \ = \ \frac{1}{-4+1}x^{-4+1} + C \ = \ -\frac{1}{3}x^{-3} + C \ = \ -\frac{1}{3x^3} + C$

(2) $\displaystyle\int x^{\frac{1}{3}} dx \ = \ \frac{1}{\frac{1}{3}+1}x^{\frac{1}{3}+1} + C \ = \ \frac{3}{4}x^{\frac{4}{3}} + C \ = \ \frac{3}{4}x\sqrt[3]{x} + C$

(3) $\displaystyle\int x^{-\frac{1}{4}} dx \ = \ \frac{1}{-\frac{1}{4}+1}x^{-\frac{1}{4}+1} + C \ = \ \frac{4}{3}x^{\frac{3}{4}} + C \ = \ \frac{4}{3}\sqrt[4]{x^3} + C$

＜ 36 ページ. 合成関数の不定積分 ＞

問 1 の解答

$$(1)\ \int e^{f(x)} \times f'(x)dx \ = \ e^{f(x)} + C$$

$$(2)\ \int \cos\left(f(x)\right) \times f'(x)dx \ = \ \sin\left(f(x)\right) + C$$

$$(3)\ \int \sin\left(f(x)\right) \times f'(x)dx \ = \ -\cos\left(f(x)\right) + C$$

問 2 の解答

(1) $\log|x^4 + 5x| + C$

(2) $\log|\sin x| + C$

(3) $e^{x^2+3x} + C$

(4) $e^{-\frac{x^2}{2}} + C$

(5) $\sin(x^4 + 3) + C$

(6) $-\cos\left(\dfrac{3}{2}x^2 - 2x\right) + C$

＜ 37ページ. 積分記号 ＞

問の解答

(1) $10t - 4.9t^2 + C$

(2) $\dfrac{4\pi r^3}{3} + C$

(3) $e^u + C$

(4) $\log|y| + C$

(5) $\sin u + C$

＜ 38 ページ. 置換積分１ ＞

問の解答

$$(1)\ \int \frac{1}{u}\,\frac{du}{dx}dx \;=\; \int \frac{1}{u}du \;=\; \log|u|+C \;=\; \log|f(x)|+C$$

$$(2)\ \int \sin(u)\frac{du}{dx}dx \;=\; \int \sin(u)du \;=\; -\cos(u)+C=-\cos\left(f(x)\right)+C$$

$$(3)\ \int u^n\frac{du}{dx}dx \;=\; \int u^n du \;=\; \frac{1}{n+1}u^{n+1}+C \;=\; \frac{1}{n+1}\Big(f(x)\Big)^{n+1}+C$$

＜ 39 ページ. 置換積分 2 ＞

問 1 の解答

(1) $$\int \frac{1}{u} \times \frac{1}{a}\, du \;=\; \frac{1}{a}\int \frac{1}{u} du \;=\; \frac{1}{a}\log|u| + C \;= \frac{1}{a}\log|ax+b| + C$$

$$\left(u = ax + b \;\Rightarrow\; x = \frac{1}{a}\, u - \frac{b}{a} \;\Rightarrow\; \frac{dx}{du} = \frac{1}{a} \;\Rightarrow\; dx = \frac{1}{a}\, du \right)$$

(2) $$\int \sin(u)\, \frac{1}{a}\, du \;=\; \frac{1}{a} \int \sin u du \;=\; -\frac{1}{a}\cos u + C \;=\; -\frac{1}{a}\cos(ax+b) + C$$

(3) $$\int u^n \frac{1}{a}\, du \;=\; \frac{1}{a}\int u^n\, du \;=\; \frac{1}{a} \times \frac{1}{n+1}\, u^{n+1} + C \;=\; \frac{1}{a(n+1)}(ax+b)^{n+1} + C$$

問 2 の解答

(1) $\dfrac{1}{4}e^{4x+5} + C$

(2) $\dfrac{1}{3}\sin(3x-5) + C$

(3) $\dfrac{1}{5}\log|5x+6| + C$

(4) $-\dfrac{1}{2}\cos(2x+\pi) + C$

(5) $\dfrac{1}{48}(8x+7)^6 + C$

(6) $-\dfrac{1}{5(5x+6)} + C$

＜ 40 ページ. 置換積分 3 ＞

問の解答

(1) $\displaystyle\int xe^u \times \frac{1}{2x}\, du \;=\; \int \frac{1}{2}e^u\, du \;=\; \frac{1}{2}e^u + C \;=\; \frac{1}{2}e^{x^2+1} + C$

$\left(u = x^2 + 1 \;\Rightarrow\; \dfrac{du}{dx} \;=\; 2x \;\Rightarrow\; dx = \dfrac{1}{2x}\, du\right)$

(2) $\displaystyle\int x^3 e^u \times \frac{1}{4x^3} du \;=\; \frac{1}{4}\int e^u du \;=\; \frac{1}{4}e^u + C \;=\; \frac{1}{4}e^{x^4} + C$

$\left(u = x^4 \;\Rightarrow\; \dfrac{du}{dx} = 4x^3\right)$

(3) $\displaystyle\frac{1}{3}\int \cos u du \;=\; \frac{1}{3}\sin(x^3+2) + C$

$(u = x^3 + 2)$

(4) $\displaystyle\frac{1}{2}\int \sin u du \;=\; -\frac{1}{2}\cos(x^2+3) + C$

$(u = x^2 + 3)$

(5) $\displaystyle\frac{1}{2}\int \frac{1}{u} du \;=\; \frac{1}{2}\log\left|x^2+3\right| + C$

$(u = x^2 + 3)$

(6) $\displaystyle\frac{1}{2}\int u^5 du \;=\; \frac{1}{2}\times\frac{1}{6}u^6 + C \;=\; \frac{1}{12}(x^2+1)^6 + C$

$(u = x^2 + 1)$