

2002年度 基礎数学ワークブック

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高知工科大学
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Series **A**

No. **8**

解答

< 1 ページ. 複素数の四則演算 1 >

問 1 の解答

$$(1) \quad (2+i) + (3-i) \\ = 5$$

$$(2) \quad (4-i) - (5-3i) \\ = -1+2i$$

$$(3) \quad \left(0.13 + \frac{1}{2}i\right) + \left(\frac{3}{4} - 1.5i\right) \\ = 0.13 + 0.75 + \left(\frac{1}{2} - \frac{3}{2}\right)i = 0.88 - i$$

$$(4) \quad \left(\frac{1}{4} - \frac{1}{3}i\right) - \left(\frac{1}{8} - \frac{1}{3}i\right) \\ = \frac{1}{8}$$

$$(5) \quad (\sqrt{3}-i) + (\sqrt{1}-2i) \\ = \sqrt{3} + 1 - 3i$$

$$(6) \quad \left(\frac{1}{4} - \sqrt{2}i\right) - \left(\frac{1}{3} + \sqrt{3}i\right) \\ = -\frac{1}{12} - (\sqrt{2} + \sqrt{3})i$$

問 2 の解答

$$(1) \quad 3(4+i) \\ = 12 + 3i$$

$$(2) \quad 6\left(\frac{1}{4} - \frac{1}{2}i\right) \\ = \frac{3}{2} - 3i$$

$$(3) \quad 3(6-2i) - 4(2-i) \\ = 18 - 6i - 8 + 4i \\ = 10 - 2i$$

$$(4) \quad \sqrt{3}\left(\frac{1}{\sqrt{3}} - \sqrt{3}i\right) + \left(\frac{1}{3} - 2i\right) \\ = 1 - 3i + \frac{1}{3} - 2i \\ = \frac{4}{3} - 5i$$

< 2 ページ. 複素数の四則演算 2 >

問の解答

(1) $i^3 = -i$

(2) $i^4 = 1$

(3) $i^5 = i$

(4) $i^6 = -1$

(5) $i^7 = -i$

(6) $i^8 = 1$

(7) $(1+i)(1-i) = 1 - i^2 = 2$

(8) $(2 + \sqrt{3}i)(2 - \sqrt{3}i) = 4 - 3i^2 = 7$

(9) $\left(\frac{\sqrt{3}+i}{2}\right)\left(\frac{\sqrt{3}-i}{2}\right) = \frac{3-i^2}{4} = 1$

(10) $(-1+i)^2 = 1 - 2i + i^2 = -2i$

(11) $(-1-i)^2 = 1 + 2i + i^2 = 2i$

(12) $(4+2i)(2-3i) = 8 - 12i + 4i - 6i^2$
 $= 14 - 8i$

(13) $(3-2i)(1-3i) = 3 - 9i - 2i + 6i^2$
 $= -3 - 11i$

(14) $(3-i)^3 = 3^3 - 3 \times 3^2i + 3 \times 3 \times i^2 - i^3$
 $= 27 - 27i - 9 - (-i)$
 $= 18 - 26i$

< 3 ページ. 複素数の四則演算 3 >

問の解答

$$(1) \frac{-1}{1+i} = \frac{-1(1-i)}{1^2-i^2} = \frac{i-1}{2} \quad (2) \frac{-1}{1-i} = \frac{-(1+i)}{1^2-i^2} = \frac{-1-i}{2}$$

$$(3) \frac{-i}{1-i} = \frac{-i(1+i)}{1^2-i^2} = \frac{-i+1}{2} \quad (4) \frac{3}{\sqrt{5}-i} = \frac{3(\sqrt{5}+i)}{5-i^2} = \frac{3(\sqrt{5}+i)}{6}$$

$$= \frac{1-i}{2} \quad = \frac{\sqrt{5}+i}{2}$$

$$(5) \frac{7}{3+\sqrt{5}i} = \frac{7(3-\sqrt{5}i)}{3^2-5i^2} \quad (6) \frac{-i}{1+i} = \frac{-i(1-i)}{1^2-i^2} = \frac{-i+i^2}{2}$$

$$= \frac{7(3-\sqrt{5}i)}{14} = \frac{3-\sqrt{5}i}{2} \quad = \frac{-1-i}{2}$$

$$(7) \frac{1}{\sqrt{3}i(\sqrt{3}+i)} = \frac{1}{3i-\sqrt{3}} \quad (8) \frac{\sqrt{2}}{\sqrt{2}-i} = \frac{\sqrt{2}(\sqrt{2}+i)}{2-i^2} = \frac{2+\sqrt{2}i}{3}$$

$$= \frac{3i+\sqrt{3}}{(3i)^2-3} = \frac{3i+\sqrt{3}}{-9-3} = -\frac{\sqrt{3}+3i}{12}$$

$$(9) \frac{1}{(\sqrt{2}-i)^2} = \frac{1}{2-2\sqrt{2}i+i^2} \quad (10) \frac{i}{(1+i)^4} = \frac{i}{1+4i+6i^2+4i^3+i^4}$$

$$= \frac{1}{1-2\sqrt{2}i} = \frac{1+2\sqrt{2}i}{1^2-(2\sqrt{2}i)^2} \quad = \frac{i}{1+4i-6-4i+1}$$

$$= \frac{1+2\sqrt{2}i}{9} \quad = -\frac{i}{4}$$

< 4 ページ. 負の数の平方根 >

問の解答

$$(1) \sqrt{(-3) \times (-4) \times (-5)}$$
$$= \sqrt{-60} = \sqrt{60}i = 2\sqrt{15}i$$

$$(2) \sqrt{-3} \times \sqrt{-4} \times \sqrt{-5} = \sqrt{3}i \times 2i \times \sqrt{5}i$$
$$= -2\sqrt{15}i$$

$$(3) \frac{\sqrt{12}}{\sqrt{-4}} = \frac{2\sqrt{3}}{2i} = \frac{\sqrt{3}i}{i^2} = -\sqrt{3}i$$

$$(4) \sqrt{\frac{12}{-4}} = \sqrt{-3} = \sqrt{3}i$$

< 5 ページ.2 次方程式 >

問の解答

$$(1) \quad x^2 + x + 2 = 0 \quad x = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{7}i}{2}$$

$$(2) \quad x^2 + 3x + 9 = 1$$

$$x^2 + 3x + 8 = 0 \quad x = \frac{-3 \pm \sqrt{9-32}}{2} = \frac{-3 \pm \sqrt{23}i}{2}$$

$$(3) \quad 3x^2 - 5x + 4 = 0 \quad x = \frac{5 \pm \sqrt{25-48}}{6} = \frac{5 \pm \sqrt{23}i}{6}$$

< 6 ページ.2 次式の因数分解 >

問の解答

$$(1) \quad x^2 - 2x + 5 = (x - 1)^2 + 4 = (x - 1 - 2i)(x - 1 + 2i)$$

$$(2) \quad -5x^2 + 4x - 3 = -5 \left(x - \frac{2 - \sqrt{11}i}{5} \right) \left(x - \frac{2 + \sqrt{11}i}{5} \right) \\ = -5 \left(x - \frac{2}{5} + \frac{\sqrt{11}i}{5} \right) \left(x - \frac{2}{5} - \frac{\sqrt{11}i}{5} \right) = -5 \left(x - \frac{2}{5} + \frac{\sqrt{11}i}{5} \right) \left(x - \frac{2}{5} - \frac{\sqrt{11}i}{5} \right)$$

$$(3) \quad 3x^2 - 3x + 3 = 3(x^2 - x + 1) = 3 \left(x - \frac{1 + \sqrt{3}i}{2} \right) \left(x - \frac{1 - \sqrt{3}i}{2} \right) \\ = 3 \left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2} \right) \left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2} \right)$$

< 7 ページ. 高次式の因数分解 >

問の解答

$$\begin{aligned} (1) \quad x^3 - 1 &= (x - 1)(x^2 + x + 1) = (x - 1) \left(x - \frac{-1 + \sqrt{3}i}{2} \right) \left(x - \frac{-1 - \sqrt{3}i}{2} \right) \\ &= (x - 1) \left(x + \frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \left(x + \frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \end{aligned}$$

$$(2) \quad x^3 + 8 = (x + 2)(x^2 - 2x + 4) = (x + 2)(x - 1 - \sqrt{3}i)(x - 1 + \sqrt{3}i)$$

$$(3) \quad x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x - i)(x + i)$$

< 8 ページ. 高次方程式 >

問の解答

$$(1) \quad x^3 - 1 = 0 \quad (x-1) \left(x - \frac{-1 + \sqrt{3}i}{2} \right) \left(x - \frac{-1 - \sqrt{3}i}{2} \right) = 0$$

$$\underline{\text{(答)} \quad x = 1, \frac{-1 + \sqrt{3}i}{2}, \frac{-1 - \sqrt{3}i}{2}}$$

$$(2) \quad x^3 + 27 = 0 \quad (x+3)(x^2 - 3x + 9) = 0$$

$$\underline{\text{(答)} \quad x = -3, \frac{3 \pm 3\sqrt{3}i}{2}}$$

$$(3) \quad x^4 - 1 = 0 \quad (x-1)(x+1)(x-i)(x+i) = 0$$

$$\underline{\text{(答)} \quad x = \pm 1, \pm i}$$

< 9 ページ. 共役複素数 >

問 1 の解答

(1) $z = 1, \bar{z} = 1$

(2) $z = i, \bar{z} = -i$

(3) $z = 1 - i, \bar{z} = 1 + i$

(4) $z = \frac{1+i}{2}, \bar{z} = \frac{1-i}{2}$

問 2 の解答

(1) $\frac{1}{2}(z + \bar{z})$

$$= 4$$

(2) $\frac{1}{2i}(z - \bar{z})$

$$\begin{aligned} &= \frac{1}{2i}(4 + 3i - (4 - 3i)) = 4^2 - 3^2 i^2 = 25 \\ &= \frac{1}{2i} \times 6i = 3 \end{aligned}$$

(3) $z\bar{z}$

問 3 の解答

(1) $\frac{1}{2}(z + \bar{z})$

$$= a$$

(2) $\frac{1}{2i}(z - \bar{z})$

$$= b$$

(3) $z\bar{z}$

$$= a^2 + b^2$$

< 10 ページ. 絶対値 >

問 1 の解答

$$\begin{array}{llll} (1) z = -1 & (2) z = 7i & (3) z = 3 + 4i & (4) z = \frac{1+i}{2} \\ |z| = 1 & |z| = 7 & |z| = 5 & |z| = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{2}}{2} \end{array}$$

問 2 の解答

$$(1) z = 4 - 3i \qquad (2) z = 1 + i$$

$$|z|^2 = 4^2 + 3^2 = 25$$

$$|z|^2 = 1^2 + 1^2 = 2$$

$$\begin{aligned} z^2 &= (4 - 3i)^2 = 16 - 24i + 9i^2 \\ &= 7 - 24i \end{aligned}$$

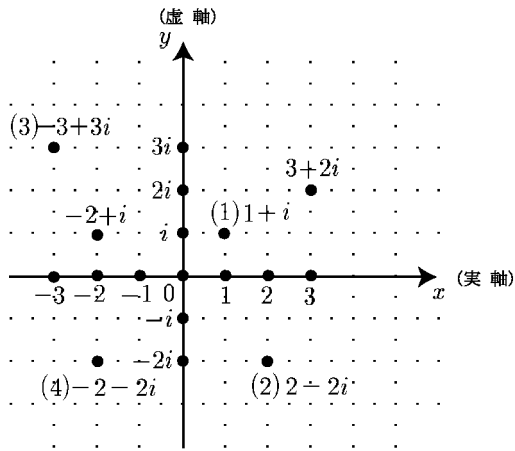
$$z^2 = (1 + i)^2 = 2i$$

$$|z^2| = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$$

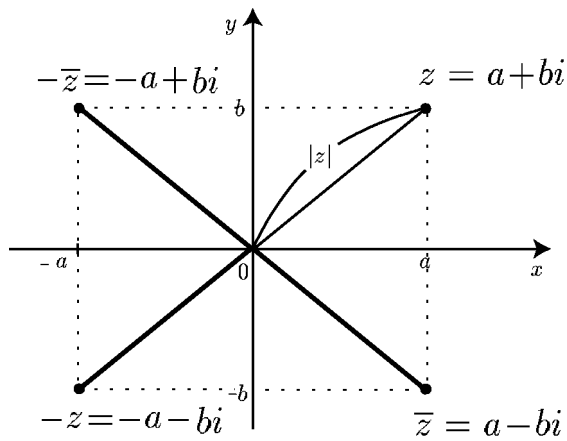
$$|z^2| = \sqrt{2^2} = 2$$

< 11 ページ. 複素平面 1 >

問1の解答



問2の解答



< 12 ページ. 複素平面 2 >

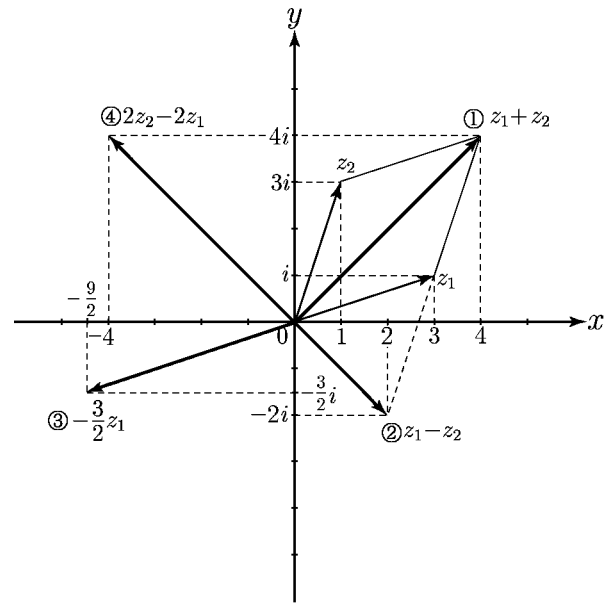
問の解答

$$\begin{aligned} z_1 + z_2 &= (3 + i) + (1 + 3i) \\ &= 4 + 4i \end{aligned}$$

$$\begin{aligned} z_1 - z_2 &= (3 + i) - (1 + 3i) \\ &= 2 - 2i \end{aligned}$$

$$\begin{aligned} -\frac{3}{2}z_1 &= -\frac{3}{2}(3 + i) \\ &= -\frac{9}{2} - \frac{3}{2}i \end{aligned}$$

$$\begin{aligned} 2z_2 - 2z_1 &= 2(z_2 - z_1) \\ &= 2\{(1 + 3i) - (3 + i)\} \\ &= 2(-2 + 2i) \\ &= -4 + 4i \end{aligned}$$



< 13 ページ. 複素数の i 倍 >

問の解答

$$(1) z = 1 + i$$

$$iz = i(1 + i) = i - 1 = -1 + i$$

$$i^2z = i(i - 1) = -1 - i$$

$$i^3z = i(-1 - i) = -i + 1 = 1 - i$$

$$i^4z = i(-i + 1) = 1 + i$$

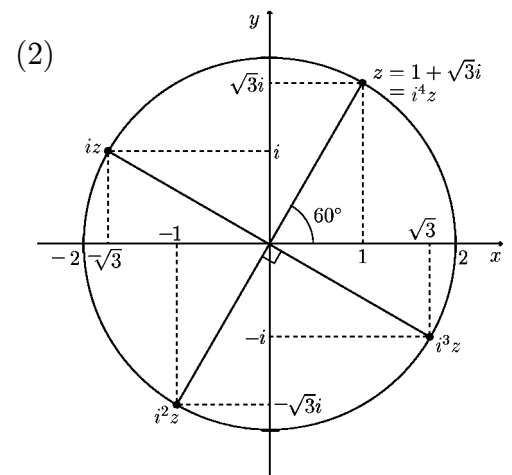
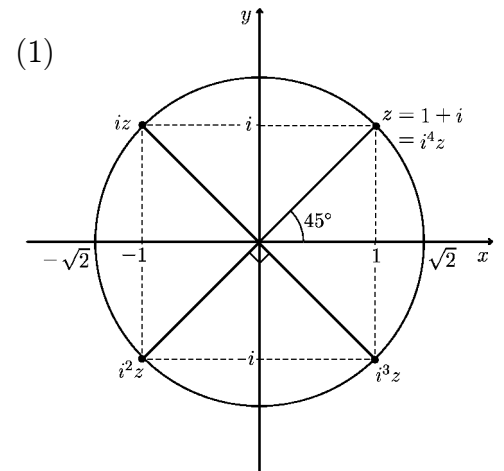
$$(2) z = 1 + \sqrt{3}i$$

$$iz = i(1 + \sqrt{3}i) = i - \sqrt{3} = -\sqrt{3} + i$$

$$i^2z = i(i - \sqrt{3}) = -1 - \sqrt{3}i$$

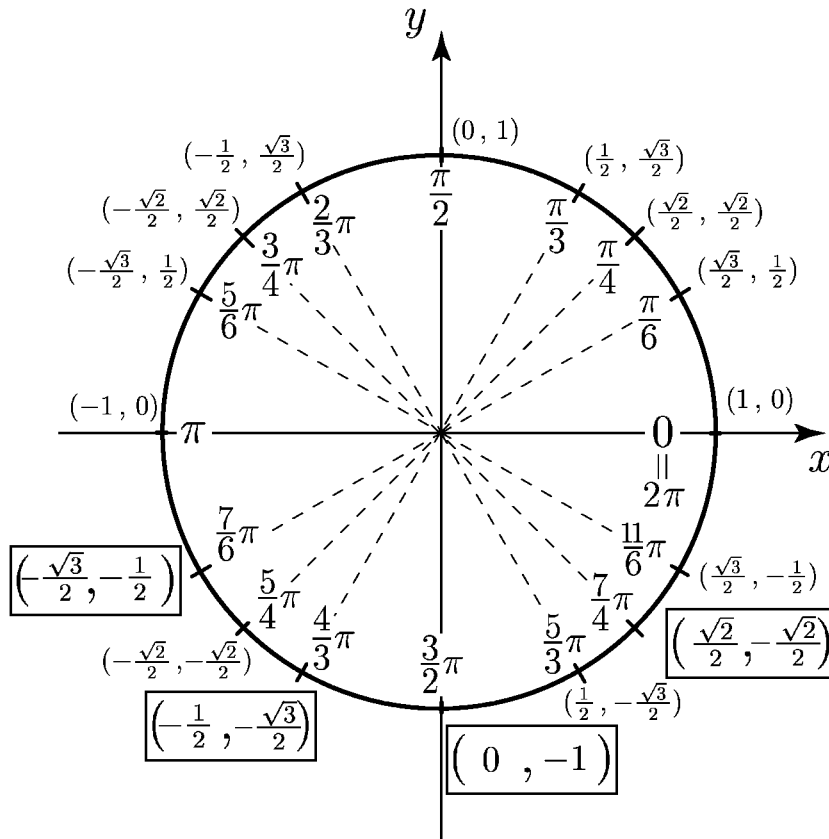
$$i^3z = i(-1 - \sqrt{3}i) = -i + \sqrt{3} = \sqrt{3} - i$$

$$i^4z = i(-i + \sqrt{3}) = 1 + \sqrt{3}i$$



< 14 ページ. 極座標 1 >

問1の解答



問2の解答

$$(1) \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \left(\cos \frac{2}{3}\pi, \sin \frac{2}{3}\pi\right)$$

$$(2) \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(\cos \frac{\pi}{4}, \sin \frac{\pi}{4}\right)$$

$$(3) (1, 0) = (\cos 0, \sin 0)$$

$$(4) \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) = \left(\cos \frac{7}{6}\pi, \sin \frac{7}{6}\pi\right)$$

$$(5) (0, -1) = \left(\cos \frac{3}{2}\pi, \sin \frac{3}{2}\pi\right)$$

$$(6) \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = \left(\cos \frac{5}{4}\pi, \sin \frac{5}{4}\pi\right)$$

< 15 ページ. 極座標表示 2 >

問の解答

$$(1) (3, 3) = \left(3\sqrt{2} \cos \frac{\pi}{4}, 3\sqrt{2} \sin \frac{\pi}{4} \right)$$

$$\begin{aligned} (2) (1, -\sqrt{3}) &= \left(2 \cos\left(-\frac{\pi}{3}\right), 2 \sin\left(-\frac{\pi}{3}\right) \right) \\ &= \left(2 \cos\left(\frac{5\pi}{3}\right), 2 \sin\left(\frac{5\pi}{3}\right) \right) \end{aligned}$$

$$(3) (\sqrt{3}, 1) = \left(2 \cos \frac{\pi}{6}, 2 \sin \frac{\pi}{6} \right)$$

$$(4) (-2, -2) = \left(2\sqrt{2} \cos\left(\frac{5}{4}\pi\right), 2\sqrt{2} \sin\left(\frac{5}{4}\pi\right) \right)$$

＜ 16 ページ. 絶対値 1 の複素数 ＞

問の解答

$$(1) \cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right), \quad (2) \cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right), \quad (3) \cos\left(\frac{2}{3}\pi\right) + i \sin\left(\frac{2}{3}\pi\right)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \qquad = \frac{1}{2} + \frac{\sqrt{3}}{2}i \qquad = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$(4) \cos\left(\frac{3}{4}\pi\right) + i \sin\left(\frac{3}{4}\pi\right), \quad (5) \cos\left(\frac{5}{6}\pi\right) + i \sin\left(\frac{5}{6}\pi\right), \quad (6) \cos(\pi) + i \sin(\pi)$$

$$= -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \qquad = -\frac{\sqrt{3}}{2} + \frac{1}{2}i \qquad = -1$$

$$(7) \cos\left(\frac{7}{6}\pi\right) + i \sin\left(\frac{7}{6}\pi\right), \quad (8) \cos\left(\frac{5}{4}\pi\right) + i \sin\left(\frac{5}{4}\pi\right), \quad (9) \cos\left(\frac{4}{3}\pi\right) + i \sin\left(\frac{4}{3}\pi\right)$$

$$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i \qquad = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \qquad = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$(10) \cos\left(\frac{3}{2}\pi\right) + i \sin\left(\frac{3}{2}\pi\right), \quad (11) \cos\left(\frac{5}{3}\pi\right) + i \sin\left(\frac{5}{3}\pi\right), \quad (12) \cos\left(\frac{7}{4}\pi\right) + i \sin\left(\frac{7}{4}\pi\right)$$

$$= -i \qquad = \frac{1}{2} - \frac{\sqrt{3}}{2}i \qquad = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

< 17 ページ. 極形式 1 >

問の解答

$$(1) 4i = 4 \left(\cos \left(\frac{\pi}{2} \right) + i \sin \left(\frac{\pi}{2} \right) \right)$$

$$(2) -2 = 2(\cos \pi + i \sin \pi)$$

$$(3) -\sqrt{2}i = \sqrt{2} \left(\cos \left(\frac{3}{2}\pi \right) + i \sin \left(\frac{3}{2}\pi \right) \right) \\ \left(= \sqrt{2} \left(\cos \left(-\frac{\pi}{2} \right) + i \sin \left(-\frac{\pi}{2} \right) \right) \right)$$

< 18 ページ. 極形式 2 >

問の解答

$$(1) z = 1 + i = \sqrt{2} \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right)$$

$$(2) z = -1 - i = \sqrt{2} \left(\cos \left(\frac{5}{4}\pi \right) + i \sin \left(\frac{5}{4}\pi \right) \right) = \sqrt{2} \left(\cos \left(-\frac{3}{4}\pi \right) + i \sin \left(-\frac{3}{4}\pi \right) \right)$$

$$(3) z = 2\sqrt{2} + 2\sqrt{2}i = 4 \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right)$$

$$(4) z = -3 - \sqrt{3}i = 2\sqrt{3} \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = 2\sqrt{3} \left(\cos \left(\frac{7\pi}{6} \right) + i \sin \left(\frac{7\pi}{6} \right) \right) \\ = 2\sqrt{3} \left(\cos \left(-\frac{5}{6}\pi \right) + i \sin \left(-\frac{5}{6}\pi \right) \right)$$

$$(5) z = -\sqrt{18} + \sqrt{6}i = 2\sqrt{6} \left(-\sqrt{\frac{18}{24}} + \sqrt{\frac{6}{24}}i \right) = 2\sqrt{6} \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) \\ = 2\sqrt{6} \left(\cos \left(\frac{5}{6}\pi \right) + i \sin \left(\frac{5}{6}\pi \right) \right)$$

< 19 ページ. 複素数の積 >

問の解答

$$(1) \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) z = \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) r(\cos \theta + i \sin \theta)$$
$$= r \left(\cos \left(\theta + \frac{\pi}{3} \right) + i \sin \left(\theta + \frac{\pi}{3} \right) \right)$$

原点を中心として反時計まわりに $\frac{\pi}{3}(= 60^\circ)$ 回転する

$$(2) \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right) z = \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) r(\cos \theta + i \sin \theta)$$
$$= r \left(\cos \left(\theta + \frac{\pi}{4} \right) + i \sin \left(\theta + \frac{\pi}{4} \right) \right)$$

原点を中心として反時計まわりに $\frac{\pi}{4}(= 45^\circ)$ 回転する

$$(3) iz = \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) r(\cos \theta + i \sin \theta)$$
$$= r \left(\cos \left(\theta + \frac{\pi}{2} \right) + i \sin \left(\theta + \frac{\pi}{2} \right) \right)$$

原点を中心として反時計まわりに $\frac{\pi}{2}(= 90^\circ)$ 回転する

< 20 ページ. 複素数の商 >

問の解答

$$\begin{aligned}
 (1) \frac{1 + \sqrt{3}i}{\sqrt{3} + i} &= \frac{2(\frac{1}{2} + \frac{\sqrt{3}}{2}i)}{2(\frac{\sqrt{3}}{2} + \frac{1}{2}i)} = \frac{\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3})}{\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6})} = \cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right) \\
 &= \cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right)
 \end{aligned}$$

$$\begin{aligned}
 (2) \frac{1 - i}{-1 + i} &= \frac{\sqrt{2}(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i)}{\sqrt{2}(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)} = \frac{\cos(\frac{7}{4}\pi) + i \sin(\frac{7}{4}\pi)}{\cos(\frac{3}{4}\pi) + i \sin(\frac{3}{4}\pi)} = \cos \pi + i \sin \pi \\
 & \qquad \qquad \qquad \left(= \cos(-\pi) + i \sin(-\pi) \right)
 \end{aligned}$$

$$\begin{aligned}
 (3) \frac{1 - i}{-\sqrt{3} + i} &= \frac{\sqrt{2}(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i)}{2(-\frac{\sqrt{3}}{2} + \frac{1}{2}i)} = \frac{\sqrt{2}(\cos(\frac{7}{4}\pi) + i \sin(\frac{7}{4}\pi))}{2 \cos(\frac{5}{6}\pi) + i \sin(\frac{5}{6}\pi)} \\
 &= \frac{\sqrt{2}}{2} \left(\cos\left(\frac{11}{12}\pi\right) + i \sin\left(\frac{11}{12}\pi\right) \right) \\
 & \left(= \frac{\sqrt{2}}{2} \left(\cos\left(-\frac{13}{12}\pi\right) + i \sin\left(-\frac{13}{12}\pi\right) \right) \right)
 \end{aligned}$$

< 21 ページ. ド・モアブルの定理 >

問の解答

$$\begin{aligned}
 (1) \quad (-\sqrt{3} + i)^3 &= \left(2 \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) \right)^3 = 2^3 \left(\cos \left(\frac{5}{6}\pi \right) + i \sin \left(\frac{5}{6}\pi \right) \right)^3 \\
 &= 8 \left(\cos \left(\frac{5}{2}\pi \right) + i \sin \left(\frac{5}{2}\pi \right) \right) = 8i
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad \left(\frac{-1 + \sqrt{3}i}{2} \right)^6 &= \left(\cos \left(\frac{2}{3}\pi \right) + i \sin \left(\frac{2}{3}\pi \right) \right)^6 \\
 &= \cos(4\pi) + i \sin(4\pi) = 1
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad \left(\frac{1-i}{2} \right)^4 &= \left(\frac{1}{\sqrt{2}} \times \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) \right)^4 = \left(\frac{1}{\sqrt{2}} \right)^4 \left(\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right)^4 \\
 &= \frac{1}{2^2} (\cos(-\pi) + i \sin(-\pi)) = -\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad \left(\frac{-1+i}{\sqrt{3}+i} \right)^{12} &= \left(\frac{\sqrt{2} \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right)}{2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right)} \right)^{12} = \left(\frac{\sqrt{2}}{2} \right)^{12} \times \left(\frac{\cos \left(\frac{3}{4}\pi \right) + i \sin \left(\frac{3}{4}\pi \right)}{\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right)} \right)^{12} \\
 &= \left(\frac{1}{\sqrt{2}} \right)^{12} \times \left(\cos \left(\frac{7}{12}\pi \right) + i \sin \left(\frac{7}{12}\pi \right) \right)^{12} \\
 &= \frac{1}{2^6} \times (\cos(7\pi) + i \sin(7\pi)) = -\frac{1}{64}
 \end{aligned}$$

< 22 ページ.1 の累乗根 >

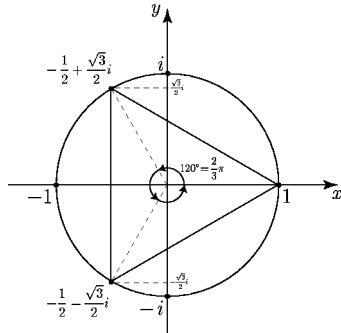
問の解答

(1) $z^3 = 1$

$$\cos(3\theta) + i \sin(3\theta) = 1$$

$$\theta = 0, \frac{2}{3}\pi, \frac{4}{3}\pi$$

$$z = 1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

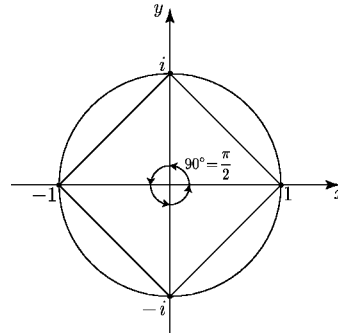


(2) $z^4 = 1$

$$\cos(4\theta) + i \sin(4\theta) = 1$$

$$\theta = 0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi$$

$$z = \pm 1, \pm i$$

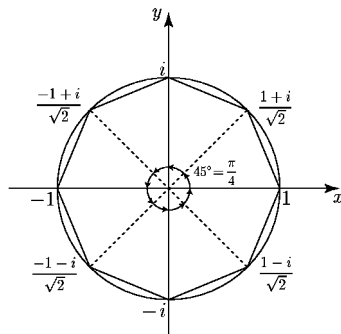


(3) $z^8 = 1$

$$\cos(8\theta) + i \sin(8\theta) = 1$$

$$\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3}{4}\pi, \pi, \frac{5}{4}\pi, \frac{3}{2}\pi, \frac{7}{4}\pi$$

$$z = 1, \frac{1+i}{\sqrt{2}}, i, \frac{-1+i}{\sqrt{2}}, -1, \frac{-1-i}{\sqrt{2}}, -i, \frac{1-i}{\sqrt{2}}$$



< 23 ページ. オイラーの公式 1 >

問の解答

$$(1) e^{2\pi i} = 1$$

$$(2) e^{-\frac{\pi}{2}i} = -i$$

$$(3) e^{\frac{3}{4}\pi i} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

$$(4) e^{\frac{5}{3}\pi i} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$(5) e^{-\frac{3}{4}\pi i} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

$$(6) e^{-\frac{\pi}{3}i} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

< 24 ページ. オイラーの公式 2 >

問の解答

(1) $e^{2-2\pi i} = e^2$

(2) $e^{0+\frac{\pi}{3}i} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

(3) $e^{2+\frac{3}{4}\pi i} = e^2 \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)$

(4) $e^{\frac{1}{2}-\frac{3}{2}\pi i} = \sqrt{e}i$

(5) $e^{\log 2 + \frac{5}{4}\pi i} = 2 \left(\cos \left(\frac{5}{4}\pi \right) + i \sin \left(\frac{5}{4}\pi \right) \right) = -\sqrt{2} - \sqrt{2}i$

(6) $e^{\frac{1}{3}\log 8 + \frac{\pi}{6}i} = 2 \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right) = \sqrt{3} + i$

< 25 ページ. 複素数の指数表示 >

問 1 の解答

$$e^{i\theta_1} \times e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$$

問 2 の解答

$$(1) e^{\frac{3}{2}\pi i} \times e^{\frac{\pi}{2}i} = e^{2\pi i} = 1$$

$$(2) e^{\frac{4}{3}\pi i} \div e^{\frac{\pi}{6}i} = e^{\frac{7}{6}\pi i} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$(3) (e^{\frac{\pi}{8}i})^4 = e^{\frac{\pi}{2}i} = i$$

$$(4) (e^{\frac{\pi}{48}i})^{12} = e^{\frac{\pi}{4}i} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

< 26 ページ. 指数法則 >

問 1 の解答

$$(2) \frac{e^{z_1}}{e^{z_2}} = e^{\boxed{z_1 - z_2}} \quad (3) (e^z)^n = e^{\boxed{nz}}$$

問 2 の解答

$$(1) e^{5+\pi i} \times e^{-1+\pi i} = e^{4+2\pi i} = e^4$$

$$(2) e^{2+\frac{\pi}{4}i} \div e^{6+\frac{\pi}{4}i} = e^{-4} = \frac{1}{e^4}$$

$$(3) \left(e^{\frac{3}{4}-\frac{3}{8}\pi i} \right)^4 = e^{3-\frac{3}{2}\pi i} = e^3 i$$

問 3 の解答

$$\frac{(1+i)^4}{(1+\sqrt{3}i)^3} = \frac{(\sqrt{2})^4 \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right)^4}{2^3 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)^3} = \frac{4 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)^4}{8 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)^3} = \frac{(\cos \pi + i \sin \pi)}{2(\cos \pi + i \sin \pi)} = \frac{1}{2}$$

< 27 ページ. 複素数の簡易表示 >

問 1 の解答

(1) $z_1 = \sqrt{3} + i$

$$= 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right)$$

$$= 2e^{\frac{\pi}{6}i}$$

(2) $z_2 = -1 + i$

$$= \sqrt{2} \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right)$$

$$= \sqrt{2}e^{\frac{3}{4}\pi i}$$

(3) $z_3 = -\sqrt{3} - 3i$

$$= 2\sqrt{3} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$

$$= 2\sqrt{3}e^{-\frac{2}{3}\pi i}$$

$$\text{または } (2\sqrt{3}e^{\frac{4}{3}\pi i})$$

問 2 の解答

(1) $z_1 z_2$

$$= 2e^{\frac{\pi}{6}i} \times \sqrt{2}e^{\frac{3}{4}\pi i}$$

$$= 2\sqrt{2}e^{\frac{11}{12}\pi i}$$

(2) $z_2 z_3$

$$= \sqrt{2}e^{\frac{3}{4}\pi i} \times 2\sqrt{3}e^{-\frac{2}{3}\pi i}$$

$$= 2\sqrt{6}e^{\frac{1}{12}\pi i}$$

$$(= 2\sqrt{6}e^{\frac{25}{12}\pi i})$$

(3) $\frac{z_3}{z_1} = \frac{2\sqrt{3}e^{-\frac{2}{3}\pi i}}{2e^{\frac{\pi}{6}i}}$

$$= \sqrt{3}e^{(-\frac{2}{3}-\frac{1}{6})\pi i}$$

$$= \sqrt{3}e^{-\frac{5}{6}\pi i}$$

$$(= \sqrt{3}e^{\frac{7}{6}\pi i})$$

< 28 ページ. 時間変数 t による微分 1 >

問の解答

$$(1) \frac{d}{dt}(9 - 6t^2 + 3t^3) = -12t + 9t^2$$

$$(2) \frac{d}{dt}(-t^8 + 3t^4 + 2t^2 + 6e^t) = -8t^7 + 12t^3 + 4t + 6e^t$$

$$(3) \frac{d}{dt}(2t^5 - 6 \cos t + \frac{1}{2} \log t) = 10t^4 + 6 \sin t + \frac{1}{2t}$$

$$(4) \frac{d}{dt} \left(\frac{5}{t} + \frac{4}{\sqrt{t^3}} \right) = -\frac{5}{t^2} - \frac{6}{t^2\sqrt{t}}$$

< 29 ページ. 時間変数 t による微分 2 >

問 1 の解答

$$(1) \frac{d}{dt} \sin(5t + 4) = 5 \cos(5t + 4)$$

$$(2) \frac{d}{dt} e^{3t+2} = 3e^{3t+2}$$

$$(3) \frac{d}{dt} \cos\left(-2t + \frac{1}{2}\right) = 2 \sin\left(-2t + \frac{1}{2}\right) \quad \left(\text{または} = -2 \sin\left(2t - \frac{1}{2}\right)\right)$$

$$(4) \frac{d}{dt} \log(9 - 2t) = \frac{-2}{9 - 2t} \left(= \frac{2}{2t - 9}\right)$$

問 2 の解答

$$(1) \frac{d}{dt} \sin(2t^3 - t) = (6t^2 - 1) \cos(2t^3 - t)$$

$$(2) \frac{d}{dt} (e^{-t^3}) = -3t^2 e^{-t^3}$$

$$(3) \frac{d}{dt} \cos(2 + 3t - 4t^2) = (8t - 3) \sin(2 + 3t - 4t^2)$$

$$\left(\text{または} - (8t - 3) \sin(4t^2 - 3t - 2)\right)$$

$$(4) \frac{d}{dt} \log(t^5 - 2t^3 + t) = \frac{5t^4 - 6t^2 + 1}{t^5 - 2t^3 + t}$$

＜ 30 ページ. 時間変数 t による微分 3 ＞

問 1 の解答

$$(1) \quad \frac{d}{dt}(2te^t) = 2e^t + 2te^t \qquad (2) \quad \frac{d}{dt}(t^3 \cos t) = 3t^2 \cos t - t^3 \sin t$$

$$(3) \quad \frac{d}{dt} \left(\frac{1}{2} e^t \sin t \right) = \frac{1}{2} e^t \sin t + \frac{1}{2} e^t \cos t \qquad (4) \quad \frac{d}{dt}(t^2 \log t) = 2t \log t + t$$

問 2 の解答

$$(1) \quad \frac{d}{dt} \left(\frac{1}{2} e^t \sin(2t) \right) = \frac{1}{2} e^t \sin(2t) + e^t \cos(2t)$$

$$(2) \quad \frac{d}{dt}(e^{3t} \cos(6t)) = 3e^{3t} \cos(6t) - 6e^{3t} \sin(6t)$$

$$(3) \quad \frac{d}{dt}(4e^{\frac{t}{2}} \sin(-5t)) = 2e^{\frac{t}{2}} \sin(-5t) - 20e^{\frac{t}{2}} \cos(-5t)$$

$$\left(\text{または } -2e^{\frac{t}{2}} \sin(5t) - 20e^{\frac{t}{2}} \cos(5t) \right)$$

$$(4) \quad \frac{d}{dt}(3e^{-2t} \cos(4t)) = -6e^{-2t} \cos(4t) - 12e^{-2t} \sin(4t)$$

< 31 ページ. 複素数値関数の微分 1 >

問の解答

$$(1) \quad z(t) = 3t^2 - 4t + (t^4 + 5t^3)i \qquad (2) \quad z(t) = e^{ibt} = \cos(bt) + i \sin(bt)$$

$$\frac{dz}{dt} = 6t - 4 + (4t^3 + 15t^2)i$$

$$\frac{dz}{dt} = -b \sin(bt) + bi \cos(bt)$$

$$(3) \quad z(t) = e^{(3+2i)t} = e^{3t} (\cos(2t) + i \sin(2t)) = e^{3t} \cos(2t) + ie^{3t} \sin(2t)$$

$$\frac{dz}{dt} = \left\{ 3e^{3t} \cos(2t) - 2e^{3t} \sin(2t) \right\} + i \left\{ 3e^{3t} \sin(2t) + 2e^{3t} \cos(2t) \right\}$$

$$(4) \quad z(t) = e^{(a+bi)t} = e^{at} (\cos(bt) + i \sin(bt)) = e^{at} \cos(bt) + ie^{at} \sin(bt)$$

$$\frac{dz}{dt} = \left\{ ae^{at} \cos(bt) - be^{at} \sin(bt) \right\} + i \left\{ ae^{at} \sin(bt) + be^{at} \cos(bt) \right\}$$

< 32 ページ. 複素数値関数の微分 2 >

問の解答

(1) $\frac{d}{dt}e^{3it} = 3ie^{3it}$

(2) $\frac{d}{dt}e^{-2it} = -2ie^{-2it}$

(3) $\frac{d}{dt}e^{bit} = bie^{bit}$

(4) $\frac{d}{dt}e^{(1+i)t} = (1+i)e^{(1+i)t}$

(5) $\frac{d}{dt}e^{(2-i)t} = (2-i)e^{(2-i)t}$

(6) $\frac{d}{dt}e^{(-3+2i)t} = (-3+2i)e^{(-3+2i)t}$

(7) $\frac{d}{dt}e^{(a-i)t} = (a-i)e^{(a-i)t}$

(8) $\frac{d}{dt}e^{(a-bi)t} = (a-bi)e^{(a-bi)t}$

< 33 ページ. 複素数の練習 1 >

問1の解答

(1) $i^7 + i^4 + i = -i + 1 + i = 1$

(2) $(i+1)(i^2 - i + 1) = i^3 + 1 = -i + 1$

(3) $\left(\frac{1+i^3}{2}\right)\left(\frac{1-i^3}{2}\right) = \left(\frac{1-i}{2}\right)\left(\frac{1+i}{2}\right) = \frac{1^2 - i^2}{4} = \frac{2}{4} = \frac{1}{2}$

(4) $\frac{1-i}{1+i} = \frac{(1-i)^2}{1^2 - i^2} = \frac{1 - 2i + i^2}{1+1} = \frac{-2i}{2} = -i$

(5) $\frac{2}{i - \sqrt{3}} = \frac{2(i + \sqrt{3})}{i^2 - 3} = \frac{2(i + \sqrt{3})}{4} = -\frac{\sqrt{3} + i}{2}$

(6) $\sqrt{-10} \times \sqrt{-6} \div \sqrt{-105} \times \sqrt{-7} = \frac{\sqrt{10}i \times \sqrt{6}i \times \sqrt{7}i}{\sqrt{105}i} = -\sqrt{\frac{420}{105}} = -2$

問2の解答

$$x = \frac{1 \pm \sqrt{1-24}}{4} = \frac{1 \pm \sqrt{23}i}{4}$$

問3の解答

$$\bar{z} = 3 - 4i$$

$$z\bar{z} = 3^2 + 4^2 = 25$$

$$|z| = \sqrt{25} = 5$$

$$z^2 = (3 + 4i)^2 = 9 + 24i + 16i^2 = -7 + 24i$$

問4の解答

(1) $3 - \sqrt{3}i = 2\sqrt{3}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

(2) $-2 + 2i = 2\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}i}\right)$

$$= 2\sqrt{3}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$$

$$= 2\sqrt{2}\left(\cos\left(\frac{3}{4}\pi\right) + i\sin\left(\frac{3}{4}\pi\right)\right)$$

$$\left(= 2\sqrt{3}\left(\cos\left(-\frac{11}{6}\pi\right) + i\sin\left(-\frac{11}{6}\pi\right)\right)\right)$$

問5の解答

(1) $\left(\frac{\sqrt{3}+i}{2}\right)^{12} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{12} = \cos(2\pi) + i\sin(2\pi) = 1$

(2) $(1-i)^8 = \left(\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)\right)^8 = (\sqrt{2})^8 \left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)^8$

$$= 2^4 (\cos(-2\pi) + i\sin(-2\pi)) = 16$$

< 34 ページ. 複素数の練習 2 >

問 1 の解答

$$(1) e^{-\frac{2\pi}{3}i} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \qquad (2) e^{3+\frac{\pi}{4}i} = e^3 \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)$$

$$(3) e^{\frac{\pi}{3}i} \div e^{\frac{\pi}{2}i} = e^{-\frac{\pi}{6}i} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

問 2 の解答

$$(1) \frac{1 - \sqrt{3}i}{2} = e^{-\frac{\pi}{3}i} \quad (\text{または } e^{\frac{5}{3}\pi i}) \qquad (2) -\frac{\sqrt{2}e}{2} + \frac{\sqrt{2}e}{2}i = e^{1+\frac{3}{4}\pi i}$$

問 3 の解答

$$(1) 1 + \sqrt{3}i = 2e^{\frac{\pi}{3}i} \qquad (2) -3 + \sqrt{3}i = 2\sqrt{3} \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = 2\sqrt{3}e^{\frac{5\pi}{6}i}$$

問 4 の解答

$$(1) e^{\frac{5\pi}{3}i} \cos\left(\frac{5}{3}\pi\right) + i \sin\left(\frac{5}{3}\pi\right) \qquad (2) e^{\frac{2+3\pi i}{4}} = e^{\frac{1}{2}} \times e^{\frac{3}{4}\pi i}$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2}i \qquad = \sqrt{e} \left\{ -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right\} \quad \left(= \frac{\sqrt{2}e}{2}(-1 + i) \right)$$

$$(3) \left(e^{\frac{\pi}{6}i} \right)^7 \div e^{\frac{4\pi}{3}i} = e^{(\frac{7}{6}\pi - \frac{4}{3}\pi)i} = e^{-\frac{\pi}{6}i} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$(4) \left(\frac{1-i}{2} \right) e^{(3+5i)t} + \left(\frac{1+i}{2} \right) e^{(3-5i)t}$$

$$= \left(\frac{1-i}{2} \right) e^{3t} (\cos(5t) + i \sin(5t)) + \left(\frac{1+i}{2} \right) e^{3t} (\cos(5t) - i \sin(5t))$$

$$= \frac{e^{3t}}{2} \left\{ (\cos(5t) + \sin(5t)) + i(-\cos(5t) + \sin(5t)) + (\cos(5t) + \sin(5t)) + i(\cos(5t) - \sin(5t)) \right\}$$

$$= e^{3t} \left\{ \cos(5t) + \sin(5t) \right\}$$

問 5 の解答

$$\cos \theta = \frac{1}{2} (e^{i\theta} + e^{-i\theta}) \qquad \sin \theta = \frac{1}{2i} (e^{i\theta} - e^{-i\theta}) \quad \left(= \frac{i}{2} (e^{-i\theta} - e^{i\theta}) \right)$$

問 6 の解答

$$(1) \frac{d}{dt} e^{t^2+t} = (2t+1)e^{t^2+t} \qquad (2) \frac{d}{dt} \{ e^{2t} \cos(3t) \} = 2e^{2t} \cos(3t) - 3e^{2t} \sin(3t)$$

$$(3) \frac{d}{dt} e^{-3ti} = -3ie^{-3ti} \qquad (4) \frac{d}{dt} e^{(4+5i)t} = (4+5i)e^{(4+5i)t}$$

< 35 ページ. 微分方程式 >

問の解答

$$(1) \frac{dy}{dt} = 2y$$

1 階微分方程式

$$(2) \frac{d^2y}{dt^2} = -9y$$

2 階微分方程式

$$(3) \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + t^4 = 0$$

3 階微分方程式

< 36 ページ. 微分方程式の解 1 >

問の解答

$$y = 3e^t$$

$$(y = -e^t \text{ など})$$

< 37 ページ. 微分方程式の解 2 >

問の解答

(1) $t = 0$ のとき $y = 3$

$$C = 3 \quad \underline{y = 3e^t}$$

(2) $t = 0$ のとき $y = -2$

$$C = -2 \quad \underline{y = -2e^t}$$

(3) $t = 0$ のとき $y = 0$

$$C = 0 \quad \underline{y = 0}$$

< 38 ページ. 微分方程式の解 3 >

問 1 の解答

$$t = 0 \text{ のとき } y = 2$$

問 2 の解答

$$y = 3e^{-t}, (y = -e^{-t} \text{ など})$$

問 3 の解答

$$y = Ce^{-t}$$

< 39 ページ. 積分の復習 >

問の解答

$$(1) \int e^y \frac{dy}{dt} dt = \int e^y dy = e^y + C$$

$$(2) \int \frac{1}{y^2} \frac{dy}{dt} dt = \int \frac{1}{y^2} dy = -\frac{1}{y} + C$$

$$(3) \int \sin y \frac{dy}{dt} dt = \int \sin y dy = -\cos y + C$$

$$(4) \int \cos y \frac{dy}{dt} dt = \int \cos y dy = \sin y + C$$

< 40 ページ. 求積法 >

問の解答

$$(1) \frac{dy}{dt} = 3t + 6$$

$$\underline{y = \frac{3}{2}t^2 + 6t + C}$$

$$(3) \frac{dy}{dt} = -\frac{2}{t^2} + \frac{1}{t}$$

$$\underline{y = \frac{2}{t} + \log t + C}$$

$$(2) \frac{dy}{dt} = \frac{1}{2}t^3 + 5t^4$$

$$\underline{y = \frac{1}{8}t^4 + t^5 + C}$$

$$(4) \frac{dy}{dt} = 4 \sin t - 5 \cos t$$

$$\underline{y = -4 \cos t - 5 \sin t + C}$$