2001年度版 基礎数学ワークブック 番外編 No.2 「データの関数近似」

高知工科大学

基礎数学ワークブック

(2001年度版)

番外編 2

「データの関数近似」

解答

＜ 番外編2「データの関数近似」　解答 ＞

p.2　＜ ラグランジュ補間 1 ＞

問 1 $y = \dfrac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0 = \dfrac{y_1(x - x_0) - y_0(x - x_0) + y_0(x_1 - x_0)}{(x_1 - x_0)}$

$= \dfrac{x - x_1}{x_0 - x_1} \times y_0 + \dfrac{x - x_0}{x_1 - x_0} \times y_1$

問 2 $p_1(x_0) = \dfrac{x_0 - x_1}{x_0 - x_1} \times y_0 + \dfrac{x_0 - x_0}{x_1 - x_0} \times y_1 = y_0$

$p_1(x_1) = \dfrac{x_1 - x_1}{x_0 - x_1} \times y_0 + \dfrac{x_1 - x_0}{x_1 - x_0} \times y_1 = y_1$

問 3 $p_2(x_0) = y_0$,　　$p_2(x_1) = y_1$,　　$p_2(x_2) = y_2$

問 4 $p_2(x) = \dfrac{(x-3)(x-4)}{(1-3)(1-4)} \times 1 + \dfrac{(x-1)(x-4)}{(3-1)(3-4)} \times (-1) + \dfrac{(x-1)(x-3)}{(4-1)(4-3)} \times 4$

$= 2x^2 - 9x + 8$

（答） $\underline{y = 2x^2 - 9x + 8}$

p.3　＜ ラグランジュ補間 2 ＞

問 1 $p_3(x_0) = y_0$, $p_3(x_1) = y_1$, $p_3(x_2) = y_2$, $p_3(x_3) = y_3$

問 2 $p_4(x_0) = y_0$, $p_4(x_1) = y_1$, $p_4(x_2) = y_2$, $p_4(x_3) = y_3$, $p_4(x_4) = y_4$

p.4　＜ ラグランジュ補間 3 ＞

$$p_4(x) = \sum_{k=0}^{4} \frac{\displaystyle\prod_{\substack{0 \leqq i \leqq 4 \\ i \neq k}} (x - x_i)}{\displaystyle\prod_{\substack{0 \leqq i \leqq 4 \\ i \neq k}} (x_k - x_i)} \times y_k$$

p.6　＜ 片対数方眼紙2 ＞

問 1 ① $y = 30 \times 10^{-\frac{1}{5}x}$　　② $y = 2 \times 10^{\frac{1}{3}(x-1)}$

問 2 ① $y = 2e^{\frac{2.3}{3}(x-1)}$　　② $y = 30e^{-\frac{2.3}{5}x}$

＜番外編2「データの関数近似」　解答＞

p.8　＜両対数方眼紙2＞

① $y = 10x$　　② $y = \dfrac{1}{100}x^2$

③ $y = 2\sqrt{x}$　　④ $y = \dfrac{100}{x}$

p.10　＜三角多項式補間2＞

(1) $f_4(0) = 1 - \cos 0 + 2\sin 0 + 3\cos 0 = 1 - 1 + 0 + 3 = 3$

(2) $f_4\left(\dfrac{\pi}{2}\right) = 1 - \cos\left(\dfrac{\pi}{2}\right) + 2\sin\left(\dfrac{\pi}{2}\right) + 3\cos\left(\dfrac{\pi}{2}\right) = 1 - 0 + 2 - 3 = 0$

(3) $f_4(\pi) = 1 - \cos\pi + 2\sin\pi + 3\cos\pi = 1 - (-1) + 0 + 3 = 5$

(4) $f_4\left(\dfrac{3}{2}\pi\right) = 1 - \cos\left(\dfrac{3}{2}\pi\right) + 2\sin\left(\dfrac{3}{2}\pi\right) + 3\cos\left(\dfrac{3}{2}\pi\right) = 1 - 0 - 2 - 3 = -4$

p.11　＜三角多項式補間3＞

$$A_2 = \frac{1}{8}\sum_{\ell=0}^{7} y_\ell \cos(2x_\ell) = \frac{1}{8}\left\{0 + 3\times 0 + 0 - 3\times 0 + 0 - 0 + 0 + 0\right\} = 0$$

$$A_4 = \frac{1}{8}\sum_{\ell=0}^{7} y_\ell \cos(4x_\ell) = \frac{1}{8}\left\{0 + 3\times(-1) + 0 - 3\times(-1) + 0 - (-1) + 0 + (-1)\right\} = 0$$

$$B_1 = \frac{1}{8}\sum_{\ell=0}^{7} y_\ell \sin(x_\ell) = \frac{1}{8}\left\{0+3\times\frac{\sqrt{2}}{2}+0-3\times\frac{\sqrt{2}}{2}+0-\left(-\frac{\sqrt{2}}{2}\right)+0+\left(-\frac{\sqrt{2}}{2}\right)\right\} = 0$$

$$B_2 = \frac{1}{8}\sum_{\ell=0}^{7} y_\ell \sin(2x_\ell) = \frac{1}{8}\left\{0 + 3\times 1 + 0 - 3\times(-1) + 0 - 1 + 0 + (-1)\right\} = \frac{1}{2}$$

$$B_3 = \frac{1}{8}\sum_{\ell=0}^{7} y_\ell \sin(3x_\ell) = \frac{1}{8}\left\{0+3\times\frac{\sqrt{2}}{2}+0-3\times\frac{\sqrt{2}}{2}+0-\left(-\frac{\sqrt{2}}{2}\right)+0+\left(-\frac{\sqrt{2}}{2}\right)\right\} = 0$$

＜番外編2「データの関数近似」　解答＞

p.12　＜三角多項式補間4＞

(1) $f_8(x_4) = f_8(\pi) = \sqrt{2}\cos\pi + \sin(2\pi) - \sqrt{2}\cos(3\pi)$

$$= \sqrt{2}\times(-1) + 0 - \sqrt{2}\times(-1) = 0$$

(2) $f_8(x_5) = f_8\left(\frac{5\pi}{4}\right) = \sqrt{2}\cos\left(\frac{5}{4}\pi\right) + \sin\left(\frac{5}{2}\pi\right) - \sqrt{2}\cos\left(\frac{15}{4}\pi\right)$

$$= \sqrt{2}\times\left(-\frac{\sqrt{2}}{2}\right) + 1 - \sqrt{2}\times\frac{\sqrt{2}}{2} = -1$$

(3) $f_8(x_6) = f_8\left(\frac{3\pi}{2}\right) = \sqrt{2}\cos\left(\frac{3}{2}\pi\right) + \sin(3\pi) - \sqrt{2}\cos\left(\frac{9}{2}\pi\right)$

$$= \sqrt{2}\times 0 + 0 - \sqrt{2}\times 0 = 0$$

(4) $f_8(x_7) = f_8\left(\frac{7\pi}{4}\right) = \sqrt{2}\cos\left(\frac{7}{4}\pi\right) + \sin\left(\frac{7}{2}\pi\right) - \sqrt{2}\cos\left(\frac{21}{4}\pi\right)$

$$= \sqrt{2}\times\frac{\sqrt{2}}{2} + (-1) - \sqrt{2}\times\left(-\frac{\sqrt{2}}{2}\right) = 1$$

p.17　＜三角多項式補間9＞

問 1　$B_k = \frac{1}{N}\sum_{\ell=0}^{N-1} y_\ell \sin\left(\frac{2\pi k\ell}{N}\right) = \frac{1}{N}\sum_{\ell=0}^{N-1} y_\ell\left\{-\sin\left(\frac{2\pi(N-k)\ell}{N}\right)\right\}$

$$= -\frac{1}{N}\sum_{\ell=0}^{N-1} y_\ell \sin\left(\frac{2\pi(N-k)\ell}{N}\right) = -B_{N-k}$$

問 2　$B_n = \frac{1}{2n}\sum_{\ell=0}^{2n-1} y_\ell \sin\left(\frac{\pi n\ell}{n}\right) = \frac{1}{2n}\sum_{\ell=0}^{2n-1} y_\ell \sin(\pi\ell) = 0$

＜番外編2「データの関数近似」　解答＞

p.20　＜離散フーリェ変換 1＞

$e^{\frac{\pi(2n-k)\ell}{n}i} = e^{-\frac{\pi k\ell}{n}i}$ とオイラーの公式及び (2) 式より

$$
\begin{aligned}
y_\ell &= A_0 + 2\sum_{k=1}^{n-1}\left\{A_k\cos\left(\frac{\pi k\ell}{n}\right) + B_k\sin\left(\frac{\pi k\ell}{n}\right)\right\} + A_n\cos(\pi\ell) \\
&= A_0 + \sum_{k=1}^{n-1}\left\{(A_k - B_k i)e^{\frac{\pi k\ell}{n}i} + (A_k + B_k i)e^{-\frac{\pi k\ell}{n}i}\right\} + A_n\cos(\pi\ell) \\
&= (A_0 - B_0 i)e^0 \ + \sum_{k=1}^{n-1}(A_k - B_k i)e^{\frac{\pi k\ell}{n}i} \\
&\qquad + \sum_{k=1}^{n-1}(A_{2n-k} - B_{2n-k} i)e^{\frac{\pi(2n-k)\ell}{n}i} + (A_n - B_n i)e^{\frac{\pi n\ell}{n}i} \\
&= \sum_{k=0}^{2n-1}(A_k - B_k i)e^{\frac{\pi k\ell}{n}i}
\end{aligned}
$$

（注）ここで和の順序を変えた等式

$$
\sum_{k=1}^{n-1}(A_{2n-k} - B_{2n-k} i)\,e^{\frac{\pi(2n-k)\ell}{n}i} = \sum_{k=n+1}^{2n-1}(A_k - B_k i)\,e^{\frac{\pi k\ell}{n}i}
$$

を使った。

p.22　＜離散フーリェ変換 3＞

$$
\begin{aligned}
C_{N-k} &= \frac{1}{N}\sum_{\ell=0}^{N-1} y_\ell e^{-\frac{2\pi(N-k)\ell}{N}i} \\
&= \frac{1}{N}\sum_{\ell=0}^{N-1} y_\ell e^{-2\pi\ell i + \frac{2\pi k\ell}{N}i} \\
&= \frac{1}{N}\sum_{\ell=0}^{N-1} y_\ell e^{\frac{2\pi k\ell}{N}i} \\
&= \overline{\frac{1}{N}\sum_{\ell=0}^{N-1} y_\ell e^{-\frac{2\pi k\ell}{N}i}} = \overline{C_k}
\end{aligned}
$$

＜ 番外編2「データの関数近似」　解答 ＞

p.26　＜ 回帰直線 3 ＞

$$y = \frac{S_{xy}}{S_{xx}}(x - \overline{x}) + \overline{y}$$

$$= \frac{5.65}{7.41}(x - 37.7) + 79.5$$