PAPER

The Optimal Calculation Method to Determine the Effective Target Width for the Application of Fitts' Law

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SUMMARY In human-computer interaction, Fitts' law has been applied in one-dimensional pointing task evaluation for some decades, and the usage of effective target width (W_e) in Fitts' law has been accepted as an international standard in ISO standards 9241-9 [4]. However, the discussion on the concrete methods for calculating W_e has not been developed comprehensively nor have the different methods of calculation been integrated. Therefore, this paper focuses on a detailed description and a comparison of the two main W_e calculation methods. One method is mapping all the abscissa data in one united relative coordinate system to perform the calculation (called CC method) and the other is dividing the data into two groups and mapping them in two separate coordinate systems (called SC method). We tested the accuracy of each method and compared both methods in a highly controlled experiment. The experiments' results and data analysis show that the CC method is better than the SC method for human computer interface modeling. These results will be instrumental for future application of Fitts' law.
key words: *human-computer interaction, Fitts' law, pointing task, effective target width*

1. Introduction

Fitts' law [3] is a famous model for one-dimensional pointing task evaluation in human computer interaction (HCI). In Fitts' law experiment, subjects are usually required to point to two rectangle targets on a platform with a pen (or on a computer monitor with a mouse) reciprocally (see Fig. 1). Then the relationship between movement time (MT) and the index of difficulty (ID_e) are described in Eq. (1), a widely used form of Fitts' law model [1], [6], [7].

$$MT = a + bID_e \tag{1}$$

$$ID_e = \log_2\left(\frac{A}{W_e} + 1\right) \tag{2}$$

A is the amplitude between the centers of two rectangular targets, and W_e is called the effective target width, which indicates the actual range of input hits around the target based on the performers' actual behavior. The Fitts' law model expressed by Eq. (1) has been accepted by ISO standards 9241-9 [4].

In Eq. (2), $W_e = 4.133SD$. SD is the standard deviation of the hits distribution.

Although the Fitts' law model defined by Eq. (1) has

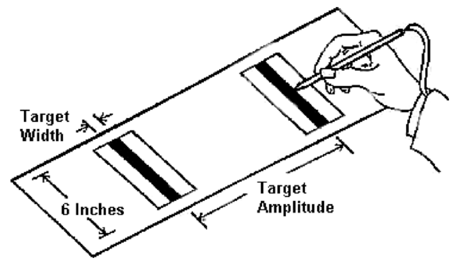

Fig. 1 Fitts' law experiment indication figure [6].

been used widely in HCI and advocated by many researchers [6], it is still not universally accepted [9]. One problem is that the calculation of W_e has not been integrated.

Here we use Fig. 2 to describe the two calculation methods of W_e. As shown in Fig. 2 (a), in a Fitts' law experiment, the subjects' input hits fall around the two rectangles. The two bell-like curves indicate the hits' distribution near the left and right rectangles. Figure 2 (b) indicates that some researchers use one united coordinate system to calculate the average of the x-coordinates to get SD and to calculate W_e, as mentioned in Douglas, Kirkpatrick and Mackenzie's research [2]. We call this method the Combined-Coordinate Method (the CC method) in this paper. Other researchers use two sets of coordinate systems to calculate the average of the x-coordinates to get SD and to calculate W_e (see Fig. 2 (c)), as Isokoski and Raisamo have done in their study [5]. In this method the averages of the x-coordinates need to be calculated for the left and right coordinate systems respectively. We call this method the Separate-Coordinate Method (the SC method) in this paper.

However, at present, no research has been reported on the preferred method of W_e calculation for the application of Fitts' law. Moreover, no comparison has been reported although the usage of W_e has been included in the ISO standards 9241-9 [4]. Therefore, in this paper we compare the two methods to see which one is better for calculating W_e. The results derived from this work will be of great help for the further application of Fitts' law to the HCI field.

2. Testing Experiment: Testing the Hits' Distribution for the SC Method

The SC method is much more complex than the CC method, but some researchers still support the SC method because they hold to the hypothesis that with bigger targets the users tend to click near the nearest edge of the rectangular target rather than near the middle of it. They therefore go on to ar-

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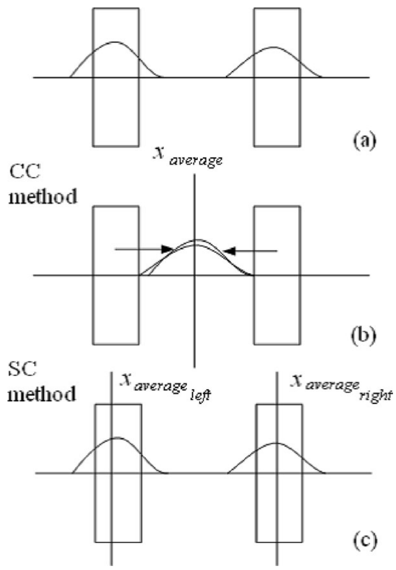

Fig. 2 The description figure of the two methods of effective target width calculation. ((a) indicates the hits distribution of the left and right targets; (b) indicates that the CC method was used to calculate the average, SD and W_e; (c) indicates that the SC method was used to calculate the average, SD and W_e with two separate coordinate systems.)

gue that if the SD is calculated in relation to a united center, the off-center click distribution will inflate the SD and bring inaccurate results of W_e [5].

To observe whether the distribution of the input hits is as Isokoski and Raisamo assumed, we developed a pointing task experiment with different A and W (target width) combinations.

2.1 Subject

Ten volunteers, five male and five female (average 28.8 years old), participated in this experiment.

2.2 Apparatus

We used a desktop PC with a color LCD monitor, the EIZO FlexScan L567 (screen size 338 mm (H) × 270 mm (V)) in this experiment. The Resolution was 1024 × 768 pixels. 1 pixel was 0.264 mm. The input device was the Microsoft Wheel Mouse Optical 1.1A.

2.3 Procedure

The experimental procedure was designed according to the ISO 9241-9 standard [4]. During the experiment, participants did reciprocal pointing with a mouse at a pair of vertical strip targets displayed on the screen. The width (W) of the targets and the center-to-center distances or amplitudes (A) between the two strips were set at W = 12, 36, 72 pixels and A =120, 360, 840 pixels. The order of the nine width and distance combinations was randomized. The start position of the cursor was the center of the screen. Twelve trials were presented in each W-A combination, with the first tap excluded in analysis. If the user tapped on the outside of the target, the task would not be abandoned and an auditory signal would be played.

The subjects were required to perform the tapping task as fast and accurately as possible, as described in Fitts' paradigm experiment [3]. During the task, except for the sound indicating a mistake had occurred, there was no other feedback to affect the subjects' performance†.

Table 1 The SD and ID_e with the CC method and the SC method in the Testing Experiment of the pointing task.

Methods	Combinations (in pixels)	SD (in pixels)	ID_e (in pixels)
the CC Method	A=120, W=12	3.44	3.24
	A=120, W=36	8.77	2.11
	A=120, W=72	12.11	1.76
	A=360, W=12	3.29	4.78
	A=360, W=36	9.73	3.31
	A=360, W=72	14.57	2.80
	A=840, W=12	3.06	6.07
	A=840, W=36	8.66	4.61
	A=840, W=72	15.32	3.83
the SC Method	A=120, W=12	3.39	3.26
	A=120, W=36	7.75	2.25
	A=120, W=72	10.07	1.96
	A=360, W=12	3.27	4.79
	A=360, W=36	9.55	3.34
	A=360, W=72	13.26	2.92
	A=840, W=12	3.05	6.08
	A=840, W=36	8.42	4.65
	A=840, W=72	15.00	3.86

2.4 Results

Table 1 shows the SD, ID_e, and the corresponding amplitude and target width combinations in the Testing Experiment.

Figures 3, 4, 5 and 6 show the input hits' distribution of the pointing task in the Testing Experiment. The abscissa values indicate the horizontal distribution range of the hits (E.g., the target width in Fig. 3 is from −18 to 18, and the target center's position is 0). The ordinate values indicate the distribution density of the hits in corresponding horizontal positions (E.g., in Fig. 3 (a), five hits fall into the area from −12 to −8, therefore, according to the regulation that we used to draw the distribution figures, the ordinate value is 5 corresponding to −8. Two hits fall into the area from −8 to −4, the ordinate value is 2 corresponding to −4.).

2.5 Discussion

Table 1 shows that the values of SD when using the SC method are less than when using the CC method, which in turn increases the values of ID_e. However the amount of change of SD is uneven. For big target sizes, the SC method decreases the SD more; for small target sizes, the SC method decreases the SD slightly or does not decrease

†In the Comparison Experiment, with each of the subjects' taps there was an instant feedback signal appearing in the screen to remind the subjects to slow down or hurry up.

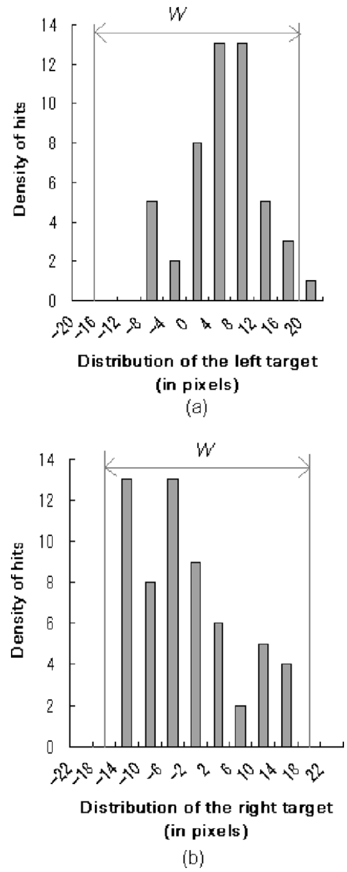

Fig. 3 The distribution of the input hits. (A=120 pixels, W=36 pixels)

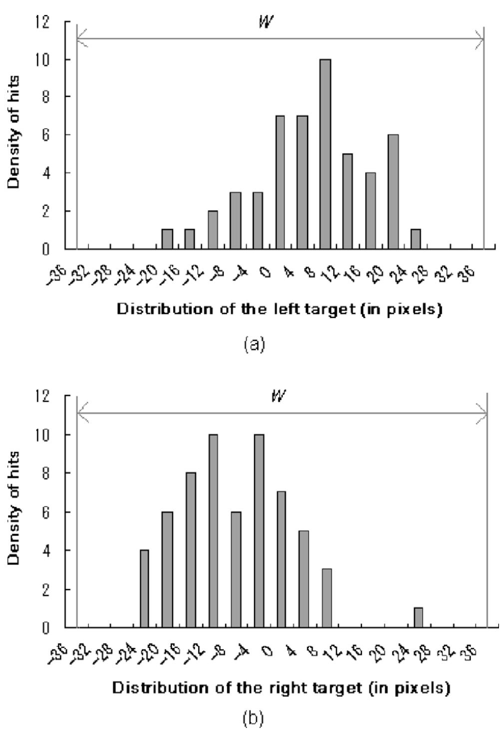

Fig. 4 The distribution of the input hits. (A=120 pixels, W=72 pixels)

the SD significantly.

We compared the effects of different target sizes in Figs. 3, 4, 5 and 6†. The off-center tendency described by

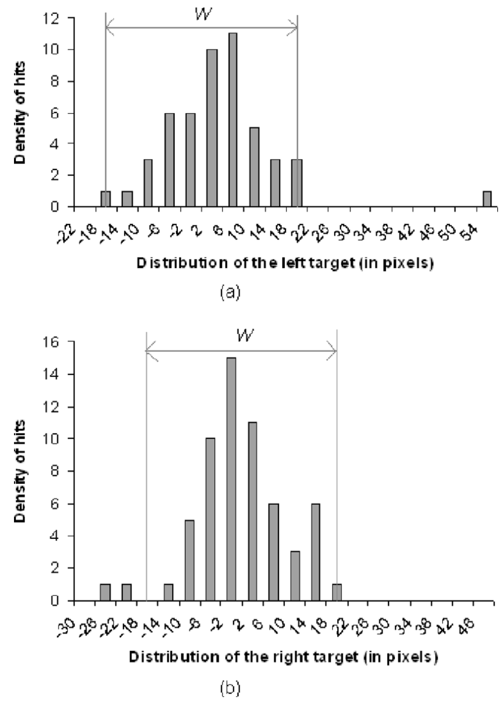

Fig. 5 The distribution of the input hits. (A=840 pixels, W=36 pixels)

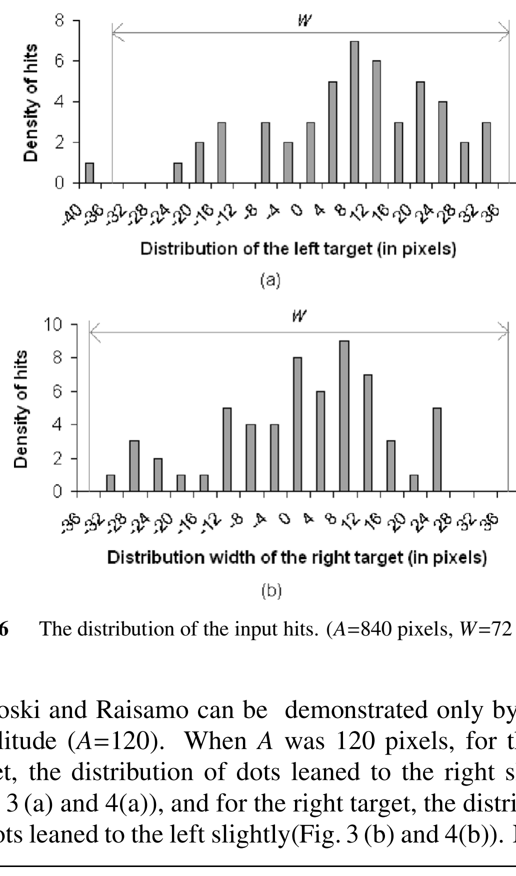

Fig. 6 The distribution of the input hits. (A=840 pixels, W=72 pixels)

Isokoski and Raisamo can be demonstrated only by small amplitude (A=120). When A was 120 pixels, for the left target, the distribution of dots leaned to the right slightly (Fig. 3 (a) and 4(a)), and for the right target, the distribution of dots leaned to the left slightly(Fig. 3 (b) and 4(b)). Never-

†Here we only give the figures of two amplitudes and two target widths due to the limited space. Since in the situation of W=12 pixels, the distributions of the input hits were limited tightly in a narrow area, the off-center inclination was not obvious, so it was unnecessary to show the distributions when W=12 pixels. The four figures shown here will be sufficient for the explanation.

Table 2 The limits of the effective target width and the corresponding appointed target width.

Appointed target width (pixels)	10	14	20	28	40
Lower limit	9.33	13.06	18.66	26.12	37.32
Upper limit	10.72	15.00	21.44	30.01	42.87

theless, with big amplitude (A=840), the off-center tendency is not clearly demonstrated with either bigger sizes (W=72 pixels) or smaller sizes (W=36 pixels): for the left target, the distribution of the dots did not lean obviously to the right of the center (Fig. 5 (a) and Fig. 6 (a)), meanwhile, the distribution of the dots around the right target did not lean obviously to the left of the center (Fig. 5 (b) and Fig. 6 (b)).

Through the Testing Experiment, we observed that the assumption of Isokoski and Raisamo was not applicable for all conditions. That meant we needed to do more work to compare the CC method and SC method. Therefore, we carried out a Comparison Experiment.

3. Comparison Experiment: Comparing the CC Method and the SC Method

Although the Testing Experiment has shown that Isokoski and Raisamo's assumption was not completely supported, a clear comparison between the CC method and SC method could not be given only through the Testing Experiment. Therefore, we were intrigued to develop another experiment to concretely check which method of W_e calculation is better.

To analyze and compare the two methods of W_e calculation accurately, we developed an experiment that could produce a set of time measurements when participants kept their tapping within the given target widths to an almost ideal extent.

Since the results would be obtained from the ideal experimental situation, we expected to see a more precisely defined difference between the two methods.

3.1 Subject

The same subjects in the Testing Experiment participated in the Comparison Experiment.

3.2 Apparatus

The same apparatus in the Testing Experiment was applied in the Comparison Experiment, but the program was different because it was designed for different experimental purposes.

3.3 Design

In the Comparison Experiment, participants reciprocally pointed with a mouse on a pair of vertical strips which were at a fixed distance apart A of 400 pixels†. W (appointed target width) was set at 10, 14, 20, 28 and 40 pixels.

If the outside region of the target was tapped, the task would not be abandoned and an auditory signal would be played as a warning signal. The start position of the cursor for both parts was the center of the screen.

We used a target width enforcement method inspired by the verbal feedback method of Zhai and colleagues [8], [10] to get the data when the subjects strictly complied with the required parameters of the program and pointed only within the target width. The purpose of this design is that by observing the ideal input hits distribution, we can see whether either of the methods is superior in modeling a pointing task.

During the experiment, if the participant took too much risk and produced a big SD and hence a big W_e, the program would remind the performer to slow down via a real-time signal which appeared on the screen. In contrast, if W_e is very small, the program would remind the participant to hurry up. If the participant's current endpoints dispersion corresponded to the ideal situation ($W = W_e$ within 7% margin) [10], no signal would appear and the participant was able to maintain his or her current pace. The judging thresholds for the different target sizes were shown in Table 2.

3.4 Procedure

We applied the following procedures for the CC method and SC method to calculate SD and control the program for the CC method and SC method.

For the CC method, the program calculated the SD based on a one coordinate system (see Fig. 2 (b)). It meant that the SD could be calculated by:

$$SD = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \tag{3}$$

In Eq. (3), x_i was the ith of the participant's selection point's x-coordinates (They were mapped into one united coordinate system). $\bar{x}$ was the mean of x-coordinates. n was the number of the trials.

For the SC method, the situation was more complex. The program calculated SD based on two sets of coordinate systems (see Fig. 2 (c)). The concrete steps were as follows: first, to compute the averages of the left and right x-coordinates of the previous 14 trials (or less than this number before the 15th trial), secondly, to get the x_i-$x_{average}$, ($i = 1, 2 \cdots n, n \leq 14$), here x_i was the ith hit's x-coordinate, and $x_{average}$ was the average of the values of x_i, then there

†In Fitts' law studies, researchers agree that amplitude plays a much less important role in pointing tasks than target width [6], [10]. In the Comparison Experiment, when we focus on the problem of distribution, it is necessary to fix the less important variables so as to simplify the problem for the purpose of comparison. Furthermore, we wanted to observe a common experimental environment in pointing tasks. So we fixed the amplitude at 400 pixels, which is about half way between the smallest value (120 pixels) and the biggest value (840 pixels) of A.

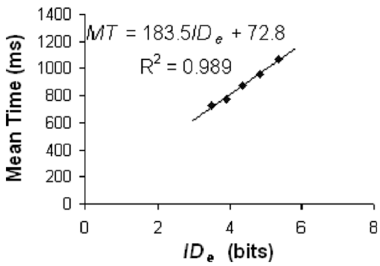

Fig. 7 The regression between mean time and ID_e using the CC method.

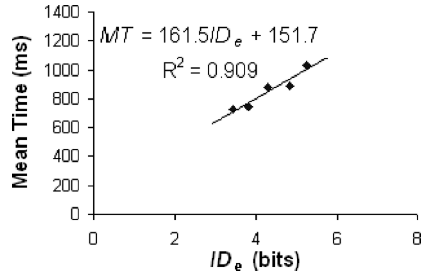

Fig. 8 The regression between mean time and ID_e using the SC method.

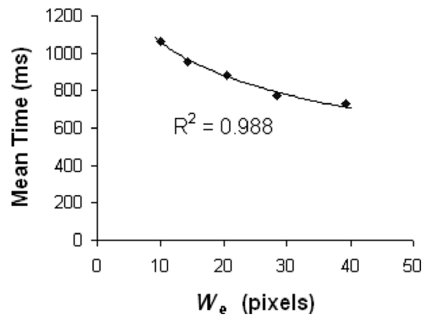

Fig. 9 The match between time and W_e by using the CC method.

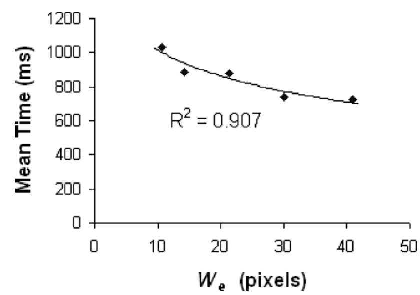

Fig. 10 The match between time and W_e by using the SC method.

should be 14 numbers of x_i-$x_{average}$. (One point noticeable here was that for the left side hits and right side hits, the values of $x_{average}$ were different†, here we used x_i-$x_{average}$ only for the convenience of the following narration. The next step was to get the SD of the 14 (x_i-$x_{average}$)s, if x'_i=x_i-$x_{average}$, then

$$SD' = \sqrt{\frac{\sum_{i=1}^{n}(x'_i - \bar{x}')^2}{n-1}} \tag{4}$$

For both the CC method and the SC method, the procedure of measuring the running W_e value was as follows: Before the participant performed the 15th trial in a W condition, the program calculated the SD of the end points based on all of the previous trials. From the 15th trial the program calculated the SD of the end points based on the immediately preceding 14 trials. The experimental program stopped the current W condition and began the next one once a block of 14 trials whose W_e matches W within a less than 7% margin was obtained. These 14 trials were used in later analysis. The program would have also aborted the current W condition if the participant had performed 30 trials without reaching a 14 trial block that met the required balance between speed and accuracy. In the actual experiment none of the participants needed to use up the maximum 30 trials.

With either W_e calculation method, the total amount of data for analysis was 700 (14 (trials) × 10 (subjects) × 5 (combinations of A and W) = 700).

3.5 Results

After the experiment, we collected data and drew the Fitts' law regression lines in Figs. 7 and 8.

In Fitts' law, the relationship between movement time and target width is a logarithm relationship (Eq. (1) and Eq. (2)). Therefore, a logarithm relation curve between movement time and W_e will be more helpful to compare the effect of the two calculation methods. Therefore, we also made the logarithmic regression lines between the MT and W_e based on the data of the experiment (Fig. 9 and Fig. 10).

3.6 Discussion

In Fig. 7, R^2 of the regression line of the CC method is near to 1 (0.989), which means that by using the CC method the regression of Fitts' law is ideal and strong. The regression of Fitts' law line in Fig. 8 is still big (0.909), but not as great as indicated by Fig. 7. This means that the SC method is not as precise as the CC method.

Fig. 9 shows that with the CC method, the logarithm relationship between movement time and effective target width is obvious and all five dots are restricted to the curve (R^2 = 0.988). However, in Fig. 10, the dots are scattered around the logarithm curve and are not confined tightly to the curve(R^2 = 0.907).

Since in the Comparison Experiment, the system gave an immediate response to the subject for each trial, the performance was under almost ideal control, therefore, the regression between MT and ID_e and the regression between MT and W_e was expected to be rather strong. From this point of view, the regression of the Fitts' law line in Fig. 8 and the logarithmic regression in Fig. 10 (related to the SC method) are not strong enough.

†For the left side hits, x_i-$x_{average}$ should be written as $x_{i_{left}}$-$x_{average_{left}}$, and for the right side hits, x_i-$x_{average}$ should be written as $x_{i_{right}}$-$x_{average_{right}}$.

4. General Discussions and Conclusions

The data from the uncontrolled Testing Experiment can help us to investigate the reason for the inadequacies in the SC method.

As explained previously, the values of W_e calculated by the SC method decrease from those values calculated by the CC method, and the changing amount for different combinations of A and W are different (see Talbe 1). For big target sizes, the SC method decreases SD and W_e more; for small target sizes, the SC method decreases SD and W_e slightly. This irregular variation of SD or W_e obtained from using the SC method will result in a weaker regression between the mean time and ID_e than the regression obtained by using the CC method. These results show that the use of the SC method produces irregular effects on different target sizes.

In the highly controlled Comparison Experiment, for the SC method, we used the two sets of coordinate systems to calculate SD, which means the requirements placed on the individual subject were less rigid than if we had used a one coordinate system. Nevertheless, when we analyze the data, we must mix all the subjects' data together, and the SD for all the dots will then be inflated. That is the reason why the effective target width obtained from the SC method is bigger than expected.

Based on the above analysis, it is logical to conclude that using one coordinate system to calculate the effective target width is more reliable.

Moreover, the CC calculation method is also much easier and more convenient than the SC method.

Another point worthy of note is that all the subjects in the two experiments were right-handed. Since for the left-handed person, the situation can simply be reversed, we can assume that the preferred hand will not affect the analytical results of this study.

In conclusion, we studied and compared two methods for calculating W_e. The results show that the CC method (Combined-Coordinate Method) is better than the SC method (Separate-Coordinate Method), i.e., it is better to map all the abscissa data into one integrated coordinate system to do the calculation, rather than to divide the data into two separate groups according to the corresponding target positions.

We believe that the data shown by this paper affords a detailed and reliable comparison of the two methods of W_e calculation based on the information derived from the input hits with different target sizes. The Combined-Coordinate method recommended in this study will help researchers and developers determine more confidently and precisely the optimum effective target widths calculation method for pointing tasks.

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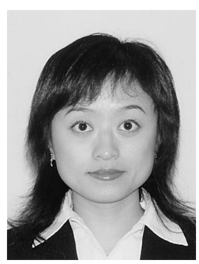

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