PERFORMANCE-BASED MINIMUM COST DESIGN OF BRIDGE SYSTEM SUBJECTED TO DEVASTATING EARTHQUAKE

Kazuhiro Taniwaki* and Kenta Tanaka**

Associate Professor, Department of Civil and Environmental Eng., Fukui University of Technology*

Chief Examiner, Chodai Co. Ltd., ootemachi 2-8-4, Naka-ku, Hiroshima**

***ABSTRACT*:** In this study, an efficient optimal performance-based seismic design method for bridge system subjected to devastating earthquakes is proposed. In the design of a bridge system, the heights of rubber bearings are taken into account as the continuous design variables, and cross-sectional dimensions and amount of steel reinforcements for RC piers and numbers of pile as the discrete design variables. The relative horizontal displacements to the both bridge and transverse directions and ductile factor are dealt with as design constraints. The construction cost minimization problem can be expressed as a mixed discrete-continuous problem, and it is solved by a classical branch and bound method with dual algorithm and convex approximation. In the optimization process, the design of experiments is applied successfully in order to calculate the dynamic behaviors and those sensitivities of the bridge system. The proposed optimal design method is applied to a five-span continuous steel girder bridge system, and it is demonstrated that the proposed method can obtain the optimum solutions quite efficiently and rigorously.

KEYWORDS: bridge system, optimum design, performance-based design, seismic design, the design of experiments

1. INTRODUCTION

After the Hyogoken Nanbu Earthquake in 1995, the seismic design code for highway bridges (Japan Road Association, 2002) has been revised in order to ensure sufficient ultimate carrying capacities in the bridge systems for large displacements caused by devastating earthquakes. Recently, the performance-based design method has been introduced for the seismic design at ultimate state. Due to recent reduction in public investment the structural engineers are highly requested to optimize the structures so as to minimize the both construction costs and maintenance costs satisfying the requested seismic performance. This task accompanies with tremendous complexity in the process of re-design of the structures for the reason of strong nonlinearity at ultimate state. Therefore, the establishment of a rational and efficient optimal seismic design method has been awaited expectantly in the practical design.

In this study, an efficient optimal performance-based seismic design method for bridge system subjected to devastating earthquakes is proposed. The bridge system consists of superstructure, rubber bearings, RC piers and RC pile foundations. In the design of a bridge system, the dimensions of superstructure are assumed to be given, and the heights of rubber bearings, cross-sectional dimensions and amount of steel reinforcements for RC piers and RC piles, and numbers of pile are taken into account as design variables. The dynamic nonlinear behaviors of the bridge system are analyzed precisely by using the general purpose nonlinear analysis software (TDAP-III) with the acceleration specified in Japanese Specifications for Highway Bridges (Japan Road Association, 2002). The relative horizontal displacements between superstructure and piers to the both bridge and transverse directions are dealt with as design constraints for the rubber bearings. The ductile factor, which is given by the ratio of working curvature to the yield curvature, is also dealt with as the design constraints for the RC piers so as to ensure the performance specified at the ultimate state. The heights of rubber bearings can take continuous values, but the other variables must be selected from discrete variable sets. Therefore, the construction cost minimization problem can be expressed as a mixed discrete-continuous problem, and it is solved by a classical branch and bound method (M.Huang, J.S. Arora, 1997) with dual algorithm and convex approximation (C. Fleury and V. Braibant, 1986) in this study. The sensitivities of the design constraints need in the optimization process and the design of experiments is applied successfully in order to calculate the dynamic behaviors and those sensitivities of the bridge system. The estimation formulae for dynamic behaviors are introduced in the expression of quadratic functions of the design variables. After the determination of optimum solution the design constraints with the estimation formulae are examined by re-analyzing the bridge system. In case that the design constraints violate the allowable limit, the estimation formulae for dynamic behaviors are improved and the minimum cost design problem is re-solved. This optimization process is iterated until the relative errors between the estimated design constrains and the exact ones satisfy the allowable limit.

The proposed optimal design method is applied to a five-span continuous steel girder bridge system, and the optimal solutions at various allowable ductility factors of RC pier are compared. In the numerical results, it is demonstrated that the reductions of the heights of rubber bearings and cross-sectional dimensions can be observed by increasing the allowable ductility factor. The optimum solutions can be obtained efficiently at a few iterations of improvements of the estimation formulae for dynamic behaviors. The accuracy of the estimation formulae is excellent within 10% relative errors between the exact behaviors and estimated ones.

2. DESIGN MODEL

In this study, the five-span continuous steel girder bridge system shown in Fig.1 is considered in which the superstructure is supported by six rubber bearings, RC piers and RC pile foundations. The

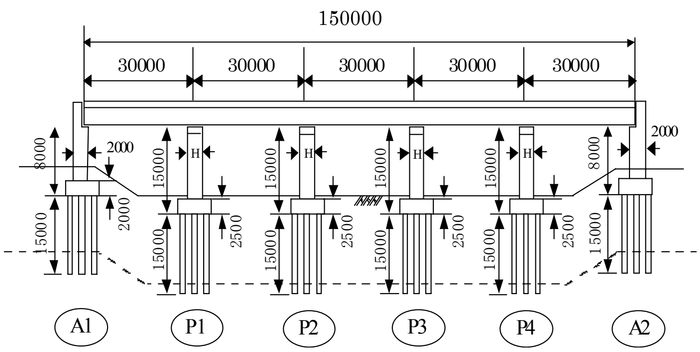

Fig.1 Five-span continuous steel girder bridge system

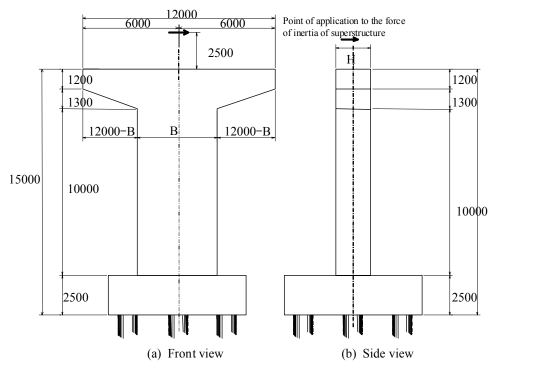

Fig.2 Front and side views of piers and RC pile foundation

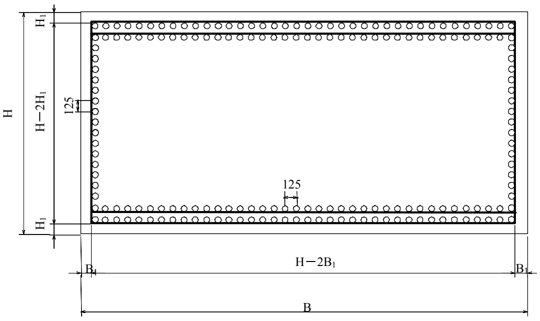

Fig.3 Cross section of a pier

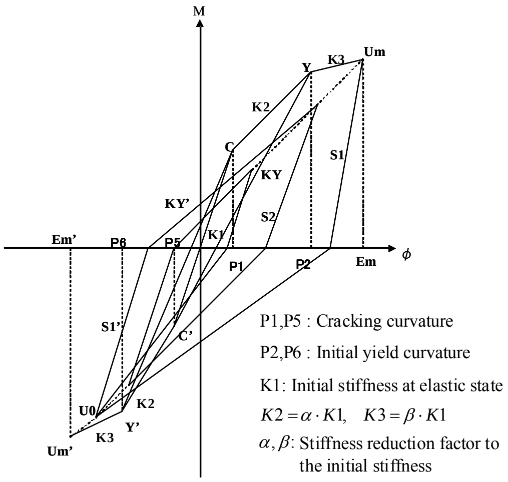

Fig.4 Trilinear rigidity hysteresis model for RC pier (Takeda model)

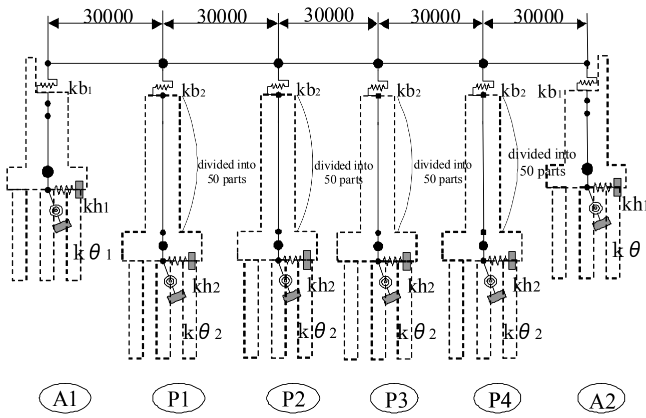

Fig.5 Nonlinear seismic response analysis model

front and side views of a pier and RC pile foundation are described in Fig.2. The lengths of piles are 15m and five types of soil conditions in stratum are considered to calculate spring constants. The reinforcements in the cross section of piers are arranged in two layers for the bridge direction and one layer for the transverse direction, and the interval of each reinforcement are fixed at 125mm as shown in Fig.3. Following an enlargement of cross sections the numbers of reinforcements increase so as to keep the intervals of reinforcements. The stiffnesses of RC piers are taken into account as the trilinear rigidity reduction type model (Takeda model) shown in Fig.4. The nonlinear seismic response analysis model of the bridge system is shown in Fig.5. The nonlinear behaviors of the bridge system for the both bridge and transverse directions subjected to devastating earthquakes are analyzed precisely by using the general purpose nonlinear analysis software (TDAP-III) in which the Type II standard strong acceleration wave motion model at the Type II soil ground specified in Japanese Specifications for Highway Bridges (Japan Road Association, 2002) is applied. In the time-history response analysis the spring constants of rubber bearings, pile foundations and superstructure are elastic, and both the superstructure and abutment are assumed as rigid body. The piers are divided into 50 segments in order to calculate the nonlinear dynamic behaviors accurately.

3. OPTIMUM DESIGN FORMULATION AND OPTIMIZATION ALGORITHM

In the design of a bridge system, the dimensions of superstructure and widths of rubber bearings are assumed to be given. The design variables for rubber bearings are the heights of those at abutment and piers, B_{h1} and B_{h2}. For RC pile foundations the numbers of piles and diameters of pile are intensively summarized as the properties of horizontal and rotation spring constants. In this study the horizontal spring constants of piles, K_h, which can be commonly used for the time-history response analysis to the both bridge and transverse directions, are considered as the design variables. The widths to the bridge and transverse directions and the amount of steel reinforcements in a cross section, H_P, B_P and A_s, are taken into account as the design variables for RC piers. The bridge system shown in Fig.1 is symmetrical to the centerline and the total number of design variables is six of $B_{h1}, B_{h2}, K_h, H_P, A_S, B_P$.

Engineers have to design the bridge systems which ensure sufficient ultimate carrying capacities for large displacements caused by devastating earthquakes. Therefore, the relative horizontal displacements between superstructure and piers to the both bridge and transverse directions are dealt with as the design constraints, $g_{h1}, g_{h2}, g_{t1}, g_{t2}$, for the safety of the rubber bearings. Furthermore, the ductile factors are also dealt with as the design constraints for the RC piers, g_{μ}, so as to ensure the performance specified at the ultimate state. The total construction cost minimization problem, which is expressed as the summation of bearing construction cost, $COST_B(B_{h1}, B_{h2})$, pier construction cost $COST_F(K_h)$ and pier construction cost $COST_P(H_P, A_S, B_P)$, can be formulated as

find $B_{h1}, B_{h2}, K_h, H_P, A_S, B_P$ which

$$\begin{aligned} \text{minimize } & COST(B_{h1}, B_{h2}, K_h, H_P, A_S, B_P) \\ & = COST_B(B_{h1}, B_{h2}) + COST_F(K_h) \\ & \quad + COST_P(H_P, A_S, B_P) \end{aligned} \tag{1}$$

subject to

$$g_{h1} = \delta_{h1} - \delta_{a1} \le 0 \tag{2}$$

$$g_{h2} = \delta_{h2} - \delta_{a2} \le 0 \tag{3}$$

$$g_{t1} = \delta_{t1} - \delta_{a1} \le 0 \tag{4}$$

$$g_{t2} = \delta_{t2} - \delta_{a2} \le 0 \tag{5}$$

$$g_{\mu} = \mu - \mu_a \le 0 \tag{6}$$

where δ_{a1} and δ_{a2} are the allowable horizontal displacements of bearings at abutment and piers, which are given as the products of the heights of bearings B_{h1}, B_{h2} multiplied by 2.5. μ is the ductile factor of a pier, which is given by the ratio of working curvature to the yield curvature for the bridge direction.

In the optimum design problem B_{h1} and B_{h2} can take continuous values, but the others must be selected from a list of discrete values. In this study, K_h, H_P, A_S and B_P are selected from the following discrete sets in which three types of piles summarized in Table 1 are considered to calculate K_h.

$K_h \in \{2514665(kN/m), 2762477, 3352886\}$

$H_P \in \{2000(mm), 2100, 2200, 2300, 2400, 2500, 2600, 2700, 2800\}$

$A_S \in \{198.6(mm^2), 286.5, 387.1, 506.7, 642.4, 794.2, 956.6, 1140\}$

$B_P \in \{3000(mm), 3500, 4000, 4500, 5000, 5500, 6000, 6500\}$

Therefore, the construction cost minimization problem can be expressed as a mixed discrete-continuous problem. Several types of optimization techniques have been developed, and Huang and Arora (M.Huang, J.S. Arora, 1997) investigated the efficiency and reliability of those for discrete and mixed discrete-continuous problems. In this study the optimization problem is solved by the classical branch and bound method with dual algorithm and convex approximation (C. Fleury and V. Braibant, 1986) for the reason that the approach is

Table 1 Properties of three types of RC piles

Diameter φ	Number of piles	Width of footing B	Width of footing H	Height of footing	Construction cost(10^3yen)	Kh(kN/m)	Kθ1(kNm/rad) (bridge direction)	Kθ2(kNm/rad) (transverse direction)	weight(kN)
1.0m	9	7.0m	7.0m	2.5m	13,466	2514665	29279320	29279320	3001.3
1.2m	9	8.4m	8.4m	2.5m	16,544	2762477	38430830	38430830	4321.8
1.0m	12	7.0m	9.5m	2.5m	17,965	3352886	57078180	39039100	4073.1

efficient and reliable for a mixed discrete-continuous problem without any parameters.

In this optimization process, in general, a number of nonlinear seismic response analyses and sensitivity analyses are necessary to determine the optimal solutions. To avoid these complexity and difficulties and make the optimum design process tremendously efficient, the design of experiments (G. Taguchi, 1987) is applied to introduce the estimation formulae for the dynamic behaviors (N. Tokunaga, S. Fukaya, C. Shen, K. Tanaka, 2001). According to the orthogonal array table $L_{27}(3^{13})$ (G. Taguchi, 1987) given in Table 2, the three levels for all design variables are assumed and the twenty seven runs of nonlinear seismic response analyses are carried out in usage of the software (TDAP-III) for the both bridge and transverse directions, respectively. The first six factors among thirty factors in Table 1 are assigned to the design variables B_{h1}, B_{h2}, K_h, H_P, A_S, B_P, respectively. Assuming that the intended variable for the kth factor is x_k and the mean value of three levels $(\hat{x}_{ki}, i=1,\cdots,3)$ for the kth factor is $\overline{x}_k$, the general form of estimation formula is introduced in the expression of quadratic functions of the design variables given as eqs.(7)-(10).

Table 2 Orthogonal array table $L_{27}(3^{13})$

	Factor												
No.	1	2	3	4	5	6	7	8	9	10	11	12	13
No.1	1	1	1	1	1	1	1	1	1	1	1	1	1
No.2	1	1	1	1	2	2	2	2	2	2	2	2	2
No.3	1	1	1	1	3	3	3	3	3	3	3	3	3
No.4	1	2	2	2	1	1	1	2	2	2	3	3	3
No.5	1	2	2	2	2	2	2	3	3	3	1	1	1
No.6	1	2	2	2	3	3	3	1	1	1	2	2	2
No.7	1	3	3	3	1	1	1	3	3	3	2	2	2
No.8	1	3	3	3	2	2	2	1	1	1	3	3	3
No.9	1	3	3	3	3	3	3	2	2	2	1	1	1
No.10	2	1	2	3	1	2	3	1	2	3	1	2	3
No.11	2	1	2	3	2	3	1	2	3	1	2	3	1
No.12	2	1	2	3	3	1	2	3	1	2	3	1	2
No.13	2	2	3	1	1	2	3	2	3	1	3	1	2
No.14	2	2	3	1	2	3	1	3	1	2	1	2	3
No.15	2	2	3	1	3	1	2	1	2	3	2	3	1
No.16	2	3	1	2	1	2	3	3	1	2	2	3	1
No.17	2	3	1	2	2	3	1	1	2	3	3	1	2
No.18	2	3	1	2	3	1	2	2	3	1	1	2	3
No.19	3	1	3	2	1	3	2	1	3	2	1	3	2
No.20	3	1	3	2	2	1	3	2	1	3	2	1	3
No.21	3	1	3	2	3	2	1	3	2	1	3	2	1
No.22	3	2	1	3	1	3	2	2	1	3	3	2	1
No.23	3	2	1	3	2	1	3	3	2	1	1	3	2
No.24	3	2	1	3	3	2	1	1	3	2	2	1	3
No.25	3	3	2	1	1	3	2	3	2	1	2	1	3
No.26	3	3	2	1	2	1	3	1	3	2	3	2	1
No.27	3	3	2	1	3	2	1	2	1	3	1	3	2

Factor1: B_{h1}, Factor2: B_{h2}, Factor3: K_h, Factor4: H_P, Factor5: A_S, Factor6: B_P

$$y = b_0 + \sum_{k=1}^{m} b_{k1} z_k + \sum_{k=1}^{m} b_{k2}(-M_{k2}^2 - M_{k3} z_k + M_{k2} z_k^2) \qquad (7)$$

where

$$M_{ki} = \frac{1}{n}(\hat{z}_{k1}^i + \hat{z}_{k2}^i + \cdots + \hat{z}_{kn}^i) \quad (k = 1,\cdots,m), \qquad (8)$$

$$\hat{z}_{ki} = \hat{x}_{ki} - \overline{x}_k \quad (i = 1,\cdots,n) \ (k = 1,\cdots,m), \qquad (9)$$

$$z_k = x_k - \overline{x}_k, \qquad (10)$$

m and n are respectively the number of factors, i.e. the number of design variables (= 6), and the number of levels for each factor (= 3). The estimation values of b_0, b_{k1} and b_{k2} in eq.(7) are given as

$$\hat{b}_0 = \frac{1}{rS_1}\sum_{i=1}^{n} T_{1i}, \quad \hat{b}_{kl} = \frac{1}{rS_k}\sum_{i=1}^{n} W_{ki} T_{ki} \quad (l = 1,2), \qquad (11)$$

where

$$S_k = \sum_{i=1}^{n} W_{ki}^2, \qquad (12)$$

r is the number of runs with the level $\hat{x}_{ki}$ (= 9). T_{ki} is the summation of results by the design of experiments with the level of $\hat{x}_{ki}$. W_{ki} is the value of function of coefficient $f_k(z_k)$ in eq.(7) with respect to b_{k1} and b_{k2} where $z_k = \hat{z}_{ki}$, namely, $W_{ki} = \hat{z}_{ki}$ and $W_{ki} = -M_{k2}^2 - M_{k3}\hat{z}_{ki} + M_{k2}\hat{z}_{ki}^2$.

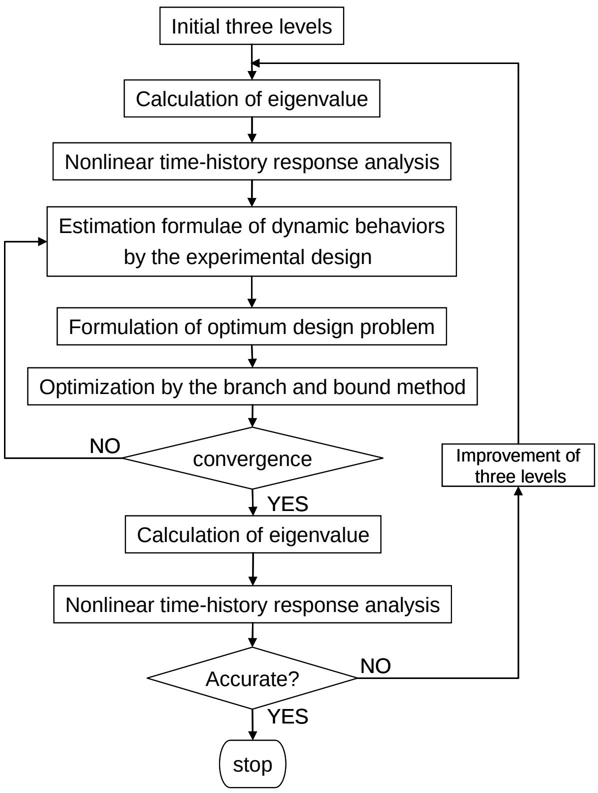

Fig.6 Macro-flow of the proposed optimum design method

Table 3 Improvements of three levels in the optimization process

Levels		B_{h1} (cm) (spring constant(kN/m))	B_{h2} (cm) (spring constant(kN/m))	spring constant of pile			H_P(mm)	As(mm^2)	B_P(mm)
				Kh(kN/m)	Kθ_1(kNm/rad)	Kθ_2(kNm/rad)			
Iteration 1	1	16.0(15313)	14.0(22857)	2514665	29279320	29279320	2400	794.4	4500
	2	14.0(17500)	12.0(26667)	2762477	38430830	38430830	2600	956.6	5000
	3	12.0(20417)	10.0(32000)	3352886	57078180	39039100	2800	1140	5500
Iteration 2	1	8.0(30625)	8.0(40000)	2514665	29279320	29279320	2400	506.7	3500
	2	10.(24500)	9.0(35556)	2762477	38430830	38430830	2600	642.4	4000
	3	12.0(20417)	10.0(32000)	3352886	57078180	39039100	2800	794.4	4500

After the determination of optimum solutions the design constraints with the estimation formulae are examined by re-analyzing the bridge system. In case that the design constraints violate the allowable limit, the three levels for all design variables and estimation formulae for dynamic behaviors are improved and the minimum cost design problem is re-solved. This optimization process is iterated until the relative errors between the estimated design constrains and the exact ones satisfy the allowable limit. The macro-flow of the proposed optimization algorithm is depicted in Fig.6.

4. DESIGN EXAMPLES

The proposed optimal design method is applied to the five-span continuous steel girder bridge system shown in Fig.1 and the optimal solutions for several allowable ductile factors μ_a are compared. In the numerical examples, the widths of bearings at abutment and each pier are assumed as 70cm and 80cm, respectively. The unit cost of rubber is as 45yen/cm^3. The construction costs of a pile are assumed as 65200yen/m^3 for the diameter 1.0m and 73800yen/m^3 for the diameter 1.2m. The construction costs of footing and form for pile foundation are assumed as 33500yen/m^3 and 8000yen/m^2. The construction costs of concrete, form and reinforcement for piers are assumed as 18500yen/m^3, 8000yen/m^2 and 120000yen/tf, respectively. Following the flow-chart in Fig.6 the optimizations for $\mu_a = 2.0$, 3.0 and 4.0 are initiated with the levels of iteration 1 shown in Table 3. In the optimization process, the lower and upper limits for discrete design variables are set at the adjacent discrete values of the minimum and maximum values of the three levels. The optimum solution for $\mu_a = 2.0$ can be obtained quite efficiently without any improvements of the three levels for all design variables. The optimum solutions for $\mu_a = 3.0$ and 4.0 determined by the lower limits. After then, the three levels are improved to the values of iteration 2 in Table 3 referring to the optimum solutions with the previous three levels. The optimum solutions for $\mu_a = 3.0$ and 4.0 can be obtained efficiently at this stage without additional improvements of the three levels. The optimum solutions for μ_a=2.0, 3.0 and 4.0 are summarized in Table 4.

In case of $\mu_a = 2.0$ the largest dimensions of cross section, H_P and B_P, and reinforcement in the piers A_S are required in order to satisfy the allowable ductile factor. By increasing the heights of rubber bearings B_{h1} and B_{h2}, namely reducing the values of spring constant, the period of bridge system is made longer and the effect from superstructure is minimized. As the result the total construction cost is minimized. In case of $\mu_a = 3.0$ B_{h1}, B_{h2}, H_P, A_S and B_P are reduced compared with those in case of $\mu_a = 2.0$. $COST_B$, $COST_P$ and the objective function $COST$ are, respectively, reduced

Table 4 Optimum solutions for $\mu = 2.0$, 3.0 and 4.0

Allowable ductile factors μ_a	2.0		3.0		4.0	
B_{h1} (spring constant)	15.3cm (16012kN/m)		11.6cm (21121kN/m)		9.0cm (27310kN/m)	
B_{h2} (spring constant)	13.0cm (24655kN/m)		9.9cm (32291kN/m)		8.0cm (40000kN/m)	
K_h	2514665kN/m		2514665kN/m		2514665kN/m	
B_P	2600mm		2800mm		2700mm	
A_s	1140mm^2		642.4mm^2		387.1mm^2	
H_P	4000mm		3500mm		3000mm	
δ_{h1}/δ_{a1}	D.exp.*	1.004	D.exp.*	0.999	D.exp.*	0.999
	Anal**	1.013	Anal**	1.006	Anal**	1.001
δ_{h2}/δ_{a2}	D.exp.*	0.824	D.exp.*	0.661	D.exp.*	0.422
	Anal**	0.834	Anal**	0.669	Anal**	0.446
δ_{t1}/δ_{a1}	D.exp.*	0.900	D.exp.*	0.961	D.exp.*	0.982
	Anal**	0.910	Anal**	0.961	Anal**	0.930
δ_{t2}/δ_{a2}	D.exp.*	1.001	D.exp.*	0.999	D.exp.*	0.921
	Anal**	1.015	Anal**	1.008	Anal**	0.824
μ/μ_a	D.exp.*	0.997	D.exp.*	1.005	D.exp.*	0.981
	Anal**	0.959	Anal**	0.988	Anal**	1.030
Total cost (10^3 yen)	192859		161254		139550	

D.exp.* : Feasibility of design constraints with the estimation formulae by the design of experiments
Anal** : Feasibility of design constraints using exact behaviors by analysis

to 24, 19 and 16 percents of those in case of $\mu_a = 2.0$. In case of $\mu_a = 3.0$ $COST_B$, $COST_P$ and $COST$ are, respectively, reduced to 39, 35 and 28 percents of those in case of $\mu_a = 2.0$. The horizontal spring constants of piles for all cases are determined by the lower limit which indicates the lowest cost and the number of piles 9..

In the investigation of active constraints at the optimum solutions, the constraints on relative horizontal displacements at abutment to the bridge direction g_{h1}, displacements at piers to the transverse direction g_{t2} and ductile factors g_μ are active for $\mu_a = 2.0$ and 3.0 simultaneously. For $\mu_a = 4.0$ B_{h2} is determined by the lower limit of 8cm and the constraints g_{h2} and g_{t2} are inactive. The maximum relative error between the exact behaviors and estimated ones is 10 percent in the constraint g_{t2} for $\mu_a = 4.0$. For the other cases the accuracy of the estimation formulae is excellent within 5 percent of relative errors. The exact constraints are enough feasible within 3 percent of violation for all cases.

5. CONCLUSIONS

The following conclusions can be drawn from this study:

1) The proposed optimal design method can determine the heights of rubber bearings, cross-sectional dimensions and amount of steel reinforcements for RC piers, and numbers of pile

rigorously and efficiently.

2) By applying the design of experiments, the estimation formulae for the ductile factor in piers and the maximum horizontal displacements to the bridge and transverse directions can be introduced accurately with small number of nonlinear seismic response analyses.
3) A few iterations of improvements for three levels are required to obtain the optimum solutions in the proposed design method.
4) In the case that the allowable ductile factor is set at a small value, the heights of rubber bearings increase in order to make the period of bridge system longer, and the effect from superstructure is minimized. As increasing the value of allowable ductile factor the heights of rubber bearings are reduced and the dimension of cross section and reinforcement in the piers are also reduced. The horizontal spring constants of piles for all cases are determined by the lower limit.
5) The constraints on relative horizontal displacements at abutment to the bridge direction, displacements at piers to the transverse direction and ductile factors are active at the optimum solutions simultaneously.

Acknowledgements

Part of this work has been supported by FUT Research Promotion Fund.

REFERENCES

C. Fleury and V. Braibant, 1986, Structural optimization: A new dual method using mixed variables, *Int. J. Num. Meth. Eng.*, 23, pp.409-428.

G. Taguchi, 1987, *System of Experimental Design*, New York, UNIPUB/Krass International Publications, Volume 1 & 2.

Japan Road Association, 2002, *Specification for Highway bridges, Part V seismic design*, Tokyo, Maruzen (in Japanese)

M.Huang, J.S. Arora, 1997, Optimal design with discrete variables: some numerical examples, *Int. J. Num. Meth. Eng.*, 40, pp.165-188.

N. Tokunaga, S. Fukaya, C. Shen, K. Tanaka, 2001, Study on the seismic behavior of steel piers with multi-plastic sections, *J. Structural Eng.*, Vol.47A, pp.819-829 (in Japanese).