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TRAJECTORY PLANNING TO IMPROVE THE MOTION PERFORMANCE FOR A NONHOLONOMIC WHEELED MOBILE ROBOT

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Abstract. *Wheeled mobile robots suitable for a variety of applications in a wide field have been studied for many years. In any application, it is necessary to ensure the motion performance of a wheeled mobile robot to guarantee its reliability and security. However, it is difficult to control the motion of wheeled mobile robot because it is a three-dimensional two-input nonholonomic system. In this paper, we address the problem of motion under nonholonomic constraints for the wheeled mobile robot based on the fact that one can direct a nonholonomic automobile to track any path in a wide space by adjusting its velocity and orientation angle on-line. First, an on-line trajectory planning method is proposed to generate the reference velocity and the orientation angle according to the target position and the robot's current position. Then, the feedback velocity and orientation control law are designed based on the dynamic model. Finally, experiments are conducted to demonstrate the effectiveness of the proposed controller.*

Keywords: Wheeled mobile robot, Noholonomic constraint, On-line trajectory planning, Motion control, Dynamics

1. Introduction.

Wheeled mobile robots suitable for a variety of applications have been studied for many years [1]. These applied mobile robots include autonomous guided vehicle (AGV) robots, the helpmate service robot that can transport food and medication throughout a predefined path, and floor-cleaning robots [2-4]. Especially, using wheeled mobile robots is an effective way to promote the factory automation. As an example, the mobile robots with intelligently motion function are used to improve the efficiency of cargo transport in factory automation. In cargo-transport task, it is essential for these robots to precisely track the predefined path from the starting location to the target location [5]. However, it is well known that the wheeled mobile robot is a nonholonomic system, because of the nonholonomic constraints, the accuracy of the path-tracking decreases and the robots stray from the predefined path, which clearly increases the danger of hitting obstacles. Therefore, when the problem of nonholonomic constraints is solved, the motion performance of the wheeled mobile robots will be further improved. The practical applications of the wheeled mobile robot will be guaranteed.

The backstepping methodology is a suitable choice to address the problem of motion for a wheeled mobile robot under nonholonomic constraints [6,7]. Recent developments include the adaptive tracking controller based on a backstepping approach [8], and a control method that enables the integration of kinematic backstepping approach and dynamic neural net works controllers [9]. However, the backstepping methodology is based on the proposition that the desired path must be continuous and has tangent direction continuity, the path curvature is continuous, and the derivative of the reference linear and angular velocity must be sufficiently small. In this paper, a trajectory planning method is proposed to address the problem of motion under nonholonomic constraints for the mobile robot. This method is based on the fact that one can direct a nonholonomic automobile to track any path in a wide space by adjusting its velocity and orientation angle on-line. Therefore, first an on-line trajectory planning method is proposed to determine the reference linear velocity and orientation angle for the robot. Then, the stability of the trajectory planning method is proven by using a Lyapunov function. In addition, a feedback controller is proposed based on the dynamic model to compute the required torque for the actual mobile robot.

This paper is organized as follows. Section 2 describes the structure of the wheeled mobile robot and derives the kinematic and dynamic model. In Section 3, an on-line trajectory planning method is presented, designed to generate the reference linear velocity and orientation angle for the steering system, to make the position error asymptotically stable. In Section 4, the feedback velocity and the orientation control law are designed based on the dynamic that the wheeled mobile robot's velocity and orientation angle converge asymptotically to the reference velocity and orientation angle inputs. Section 5 shows an experiment of the proposed method. The experimental results show that the proposed control method is feasible and effective. A conclusion is given in Section 6.

2. **Wheeled Mobile Robot and Modeling.** The wheeled mobile robot shown in Figure 1 was developed. It consists of a vehicle with two driving wheels mounted on the same axis, and two free omni-directional wheels at the front and back of the vehicle. The robot can only move in the direction normal to the axis of the driving wheels. This motion is well known as the nonholonomic constraint. The physical parameters of the wheeled mobile robot are given in Table 1.

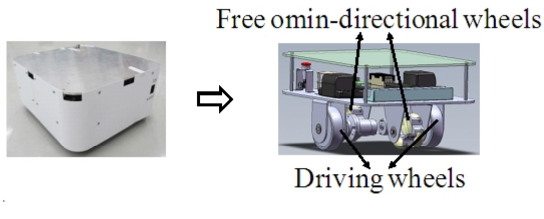

FIGURE 1. Wheeled mobile robot

TABLE 1. Physical parameters of the wheeled mobile robot

Symbol	Quantity	Value and Unit	Symbol	Quantity	Value and Unit
H	height	0.206 m	M	mass	14 kg
$2b$	width	0.300 m	m	maximum load	60 kg
$2a$	length	0.400 m	I	inertia of mass	0.28 kg·m^2

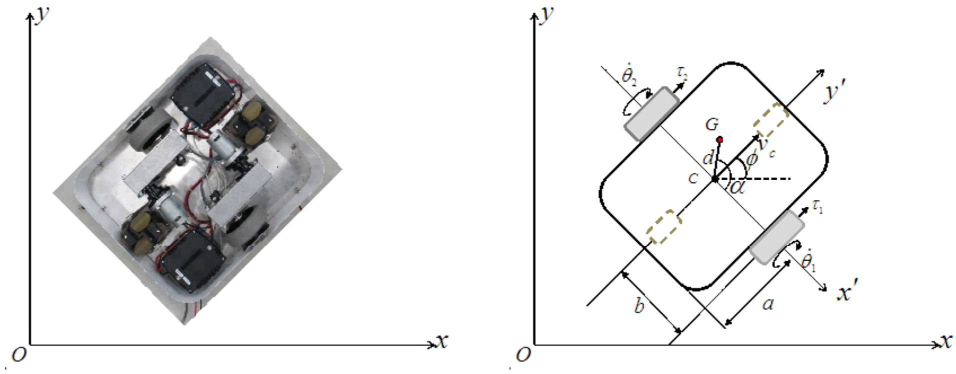

FIGURE 2. Structural model of the mobile robot

To derive the control law for the wheeled mobile robot, a schematic structure of the wheeled mobile robot is shown in Figure 2.

The parameters and coordinate systems are defined as follows: $\Sigma(x, y, O)$: absolute coordinate system; $\Sigma(x', y', C)$: translational coordinate system determined by the orientation of the mobile robot; $G(x_g, y_g)$: position of the center of gravity considering the effect of loads; $C(x_c, y_c)$: position of the geometric center; d: distance between the center of gravity and the geometrical center; v_c: linear velocity at point C; ϕ: mobile robot orientation defined as the angle between the direction of v_c and the x-axis; τ_1, τ_2: driving torques of the two driving wheels; α: angle between Cx' and CG; $2b$: distance between the driving wheels; $2a$: length of the robot; $\dot{\theta}_1, \dot{\theta}_2$: angular speeds of the two driving wheels.

First, a kinematic analysis of the nonlinear system is carried out by using the coordinate system shown in Figure 2. The kinematic equations are

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ \dot{\phi} \end{bmatrix} \tag{1}$$

The relationship between the angular speed of the two driving wheels and the robot's linear velocity and angular velocity is given as follows:

$$\begin{bmatrix} v_c \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{2}$$

where r is the wheel radius.

The Lagrange formalism is used to derive the dynamic equations for the wheeled mobile robot. In this case, the potential energy is zero because the path of the wheeled mobile robot is constrained to the horizontal plane. Considering center-of-gravity shifts and load changes caused by the applied loads, the kinetic energy of the robot is as follows:

$$E = \frac{(M+m)}{2}(\dot{x}_g^2 + \dot{y}_g^2) + \frac{I + d^2(M+m)}{2}\dot{\phi}^2 + \frac{J_\omega + k^2 J_0}{2}(\dot{\theta}_1^2 + \dot{\theta}_2^2) \tag{3}$$

where M is the robot mass and m is the mass of the applied load. Furthermore, d, which varies with the applied load's position, is the distance between the center of gravity and the geometrical center. I is the moment of inertia of the robot, J_ω is the moment of inertia of the driving wheels, J_0 is the moment of inertia of the DC motor, and k is the gear ratio.

According to the Lagrange formalism, and considering kinematic Equations (1) and (2), the complete dynamics consist of the kinematic steering system derived as follows:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \frac{m_{11}+m_{12}}{r} & \frac{m_{11}-m_{12}}{r}b \\ \frac{m_{21}+m_{22}}{r} & \frac{m_{21}-m_{22}}{r}b \end{bmatrix} \begin{bmatrix} \dot{v}_c \\ \ddot{\phi} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \dot{v}_c \\ \ddot{\phi} \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} m_{11} &= \frac{r^2(M+m)}{4b^2}(b+d\cos\alpha)^2 + \frac{r^2[I+d^2(M+m)]}{4b^2} + (J_\omega + k^2 J_0) \\ m_{12} &= \frac{r^2(M+m)}{4b^2}(b+d\cos\alpha)(b-d\cos\alpha) - \frac{r^2[I+d^2(M+m)]}{4b^2} \\ m_{21} &= m_{12} \\ m_{22} &= \frac{r^2(M+m)}{4b^2}(b-d\cos\alpha)^2 + \frac{r^2[I+d^2(M+m)]}{4b^2} + (J_\omega + k^2 J_0). \end{aligned}$$

3. **On-line Trajectory Planning for Reference Velocity and Orientation Angle.** To make the robot converge asymptotically to a desired path, first the reference linear velocity and the orientation angle must be determined for the steering system (Equation (1)). Many approaches exist to select a velocity and an orientation angle for the steering system. In this section, a simple trajectory planning method is designed to generate the reference linear velocity and the orientation angle according to the experience of steering a car.

To move the robot from the current position to the desired position, we must continue adjusting its linear velocity and orientation angle. As shown in Figure 3, in time t, the robot aims to reach the target position $[x_{cd}(t), y_{cd}(t)]$ from the current position $[x_c(t), y_c(t)]$.

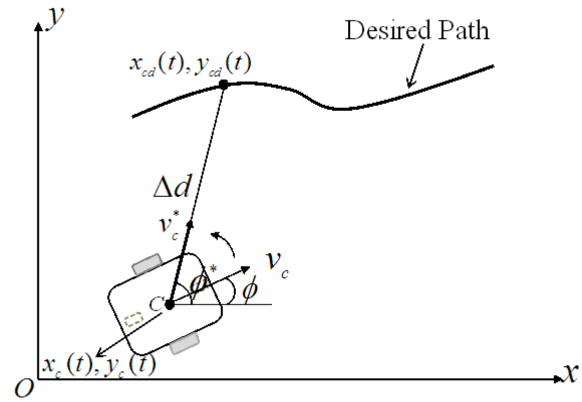

Figure 3. Design principle of target velocity and orientation angle

The robot must be oriented in the direction of the target position from the current position. Therefore, the reference orientation angle at time t is calculated as

$$\phi^* = \arctan \frac{y_{cd}(t) - y_c(t)}{x_{cd}(t) - x_c(t)} \tag{5}$$

From Equation (5), it is observed that the reference orientation angle using this calculation method is the same as the direction of the tangent of the desired path, which avoids the nonholonomic constraint on the robot.

Then, we determine the moving velocity according to the deviation distance between the current and target locations. The deviation distance is calculated as

$$\Delta d = \sqrt{[x_{cd}(t) - x_c(t)]^2 + [y_{cd}(t) - y_c(t)]^2} \tag{6}$$

In an ideal situation, the robot needs move from the current position to the target position instantly. However, the robot velocity is limited such that $|v_c| \leq V_{\max}$, where $V_{\max}$ is the maximum linear velocity of the mobile robot. Therefore, by taking the given linear velocity and the deviation distance into consideration, the target linear velocity is calculated as follows:

$$v_c^* = v_{cd} + k_v \Delta d \tag{7}$$

where k_v is a parameter to prevent the velocity becoming larger than $V_{\max}$, and $v_{cd} = \sqrt{[\dot{x}_{cd}(t)]^2 + [\dot{y}_{cd}(t)]^2}$ denotes the given linear velocity.

The stability analysis of the position tracking is given as follows. First, according to Equations (1) and (5), the derivative of deviation distance (6) is obtained as follows:

$$\begin{aligned} \Delta\dot{d} &= \frac{[x_{cd}(t) - x_c(t)][\dot{x}_{cd}(t) - \dot{x}_c(t)] + [y_{cd}(t) - y_c(t)][\dot{y}_{cd}(t) - \dot{y}_c(t)]}{\sqrt{[x_{cd}(t) - x_c(t)]^2 + [y_{cd}(t) - y_c(t)]^2}} \\ &= \cos\phi^*(v_{cd}\cos\phi_d - v_c^*\cos\phi^*) + \sin\phi^*(v_{cd}\sin\phi_d - v_c^*\sin\phi^*) \\ &= v_{cd}\cos(\phi^* - \phi_d) - v_c^*\cos(\phi^* - \phi^*) \\ &= v_{cd}\cos(\phi^* - \phi_d) - v_c^* \\ &= v_{cd}[\cos(\phi^* - \phi_d) - 1] - k_v\Delta d \end{aligned} \tag{8}$$

where $\phi_d = \arctan[\dot{y}_{cd}(t)/\dot{x}_{cd}(t)]$ is the given orientation angle.

Proof: Let us consider the Lyapunov function as follows:

$$V = \frac{1}{2}\Delta d^2 \tag{9}$$

Clearly, if $\Delta d = 0$, $V = 0$, and if $\Delta d \neq 0$, $V > 0$. Furthermore, from Equation (8), the derivative of the Lyapunov Function (9) is obtained as follows:

$$\begin{aligned} \dot{V} &= \Delta d \Delta\dot{d} \\ &= \Delta d\left\{v_{cd}[\cos(\phi^* - \phi_d) - 1] - k_v\Delta d\right\} \\ &= \Delta d \cdot v_{cd}[\cos(\phi^* - \phi_d) - 1] - k_v\Delta d^2 \end{aligned} \tag{10}$$

In Equation (10), Δd and v_{cd} are always positive, and $-2 \leq \cos(\phi^* - \phi_d) - 1 \leq 0$. Therefore, $\Delta d \cdot v_{cd}[\cos(\phi^* - \phi_d) - 1] \leq 0$. Here, when the control parameter k_v is set to be positive, $-k_v\Delta d^2 \leq 0$ and $\dot{V} \leq 0$ are obtained. Here, $\dot{V} = 0$ means $\Delta d = 0$, which implies $x_{cd}(t) - x_c(t) = 0$, and $y_{cd}(t) - y_c(t) = 0$. Since all the signals in the closed-loop system are bounded and Δd converges to zero as $t \to \infty$, the designed system is stable. Therefore, when using the path generation method of Equations (5) and (7), k_v is designed to ensure that $k_v > 0$. Then, we obtain the following equations:

$$\begin{aligned} &\lim_{t\to\infty} \Delta d = \lim_{t\to\infty} \sqrt{[x_{cd}(t) - x_c(t)]^2 + [y_{cd}(t) - y_c(t)]^2} = 0 \\ &\lim_{t\to\infty} [x_{cd}(t) - x_c(t)] = \lim_{t\to\infty} [y_{cd}(t) - y_c(t)] = 0 \end{aligned} \tag{11}$$

Therefore, the path tracking system is asymptotically stabilized to the desired path.

4. **Controller Design.** The reference linear velocity and the orientation angle in Section 3 are designed to make the position error asymptotically stable. In this section, a nonlinear feedback acceleration control method is proposed. This controller takes into account the

specific vehicle dynamics to convert a steering system command into the torque inputs for the actual vehicle. The control method is given as

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \mathbf{M} \begin{bmatrix} \dot{v}_c^* + k_{p1}(v_c^* - v_c) \\ \ddot{\phi}^* + k_{p2}(\phi^* - \phi) + k_{d2}(\dot{\phi}^* - \dot{\phi}) \end{bmatrix} \tag{12}$$

where k_{p1}, k_{p2}, and k_{d2} are control parameters of the feedback acceleration controller, $\mathbf{M}$ is the inertia matrix of the robot shown in the dynamic model (4).

The stability analysis of the control law (12) is given as follows. First, let the tracking error of v_c and ϕ be set in Equation (13):

$$e_1(t) = v_c^*(t) - v_c(t), \ e_2(t) = \phi^*(t) - \phi(t) \tag{13}$$

Substituting the control law (12) and Equation (13) into dynamic Equation (4) yields the following equation:

$$\dot{e}_1(t) + k_{p1}e_1(t) = 0, \ \ddot{e}_2(t) + k_{d2}\dot{e}_2(t) + k_{p2}e_2(t) = 0 \tag{14}$$

Here, the values of k_{p1}, k_{p2}, and k_{d2} are always set as a positive constant. Then, we obtain the following equations:

$$\lim_{t\to\infty} e_1(t) = \lim_{t\to\infty} [v_c^*(t) - v_c(t)] = 0, \ \lim_{t\to\infty} e_2(t) = \lim_{t\to\infty} [\phi^*(t) - \phi(t)] = 0 \tag{15}$$

When using the control method of Equation (12), the stability of the velocity and the orientation angle tracking can be ensured by appropriate selection of k_{d2}, k_{p1}, and k_{p2}.

5. **Experiment.** This section describes experiments conducted with the proposed controller to verify the effectiveness of the proposed algorithm. In general, the predefined path for the wheeled mobile robot to track consists of a series of straight line segments and arc line segments. It is obvious that the straight line segment is easy to track; in contrast, the arc line segment is difficult to track. To verify its effectiveness, the robot is assumed to follow a semi-circular path that is an arc line.

The path to be followed is described by

$$(x_{cd} - x_0)^2 + (y_{cd} - y_0)^2 = r^2 \tag{16}$$

where $(x_0, y_0) = (0 \text{ m}, 1 \text{ m})$ specifies the center of the semi-circle, the radius $r = 1$ m.

The trajectory is given as follows:

$$\begin{cases} x_{cd}(t) = r\cos\left[-\frac{\pi}{2} + \sigma(t)\right] + x_0 \\ y_{cd}(t) = r\sin\left[-\frac{\pi}{2} + \sigma(t)\right] + y_0 \end{cases}, \ \sigma(t) = \begin{cases} \frac{2\pi}{t_0^2}t^2 & 0 \le t \le \frac{t_0}{2} \\ \pi - \frac{2\pi}{t_0^2}(t - t_0)^2 & \frac{t_0}{2} \le t \le t_0 \end{cases} \tag{17}$$

where t_0 is the experiment execution time, which can be changed to adjust the moving speed of the robot. Here, t_0 is set to 40 s. The parameters of the proposed control method are adjusted by manually selecting the best tracking results. The selected control parameters are given as $k_{p1} = 8$ [1/s], $k_{p2} = 6.2$ [1/s^2], $k_{d2} = 3.5$ [1/s], and $k_v = 12$. Figure 4 shows the path tracking ability of the robot by the proposed method. The dashed blue line represents the reference response, and the solid red line represents the result achieved by the robot. Figure 4(a) shows the path tracking result, Figures 4(b) and 4(c) show the trajectory tracking results of the x and y positions. Clearly, by using the proposed controller, the wheeled mobile can successfully track a semi-circular path.

6. **Conclusion.** In this paper, an on-line trajectory planning method is proposed to adjust the reference velocity and the orientation angle according to the target and current position of the wheeled mobile robot, for a robot moving under nonholonomic constraints. Then, feedback acceleration control is designed based on the dynamic model to realize the required torque for the actual mobile robot. Finally, experiments are conducted and the results demonstrate the effectiveness of the proposed controller.

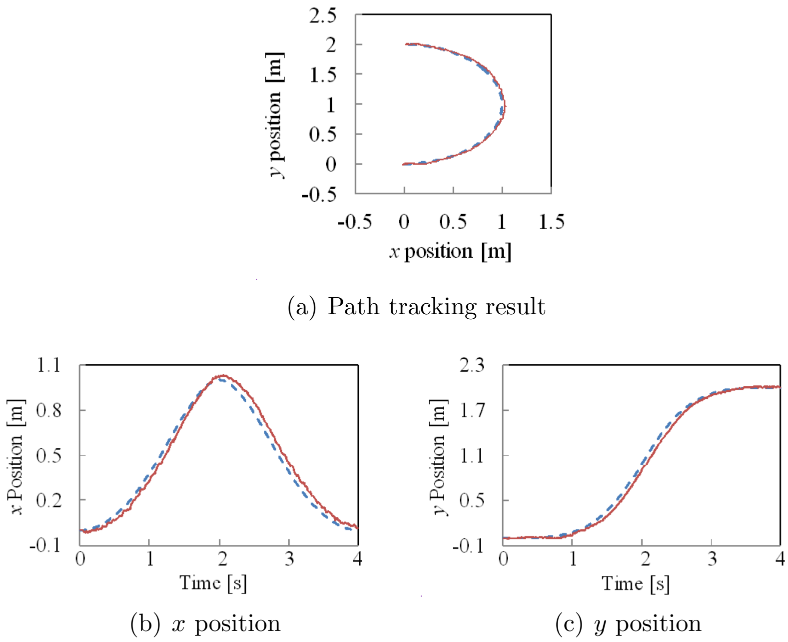

FIGURE 4. Experimental results using the proposed control method
- - reference — response

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