高知工科大学

基礎数学ワークブック

(2002 年度版)

Series A

No. 6

解答

＜ 1 ページ. 部分積分法 1 ＞

問の解答

$$\int f(x) \times g'(x)dx = f(x) \times g(x) - \int f'(x) \times g(x)dx$$

＜ 2 ページ. 部分積分法 2 ＞

問の解答

(1) $$\begin{aligned}\int (3x-2)\sin x dx &= \int (3x-2)\times(-\cos x)' dx \\ &= -(3x-2)\cos x - \int 3(-\cos x)dx \\ &= -(3x-2)\cos x + 3\sin x + C\end{aligned}$$

(2) $$\int xe^x dx = \int x \times (e^x)' dx = xe^x - \int 1\times e^x dx = xe^x - e^x + C$$

(3) $$\int (x^2+1)\cos x dx = \int (x^2+1)(\sin x)' dx = (x^2+1)\sin x - \int 2x\sin x dx$$

$$\left(\int 2x\sin x dx = 2x(-\cos x) - \int 2(-\cos x)dx = -2x\cos x + 2\sin x + C\right)$$

よって $\displaystyle\int (x^2+1)\cos x dx = (x^2+1)\sin x + 2x\cos x - 2\sin x + C$

(4) $$\begin{aligned}\int (\log x)\times x dx &= \int (\log x)\times\left(\frac{x^2}{2}\right)' dx = (\log x)\times\frac{x^2}{2} - \int \frac{1}{x}\times\frac{x^2}{2}dx \\ &= \frac{x^2}{2}\log x - \frac{1}{4}x^2 + C\end{aligned}$$

＜ 3 ページ. 三角関数の不定積分 ＞

問の解答

(1) $\displaystyle\int \sin^2 x dx = \int \left\{\frac{1}{2} - \frac{1}{2}\cos(2x)\right\} dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

(2) $\displaystyle\int \cos(3x)\cos(2x) dx = \int \left\{\frac{1}{2}\cos(5x) + \frac{1}{2}\cos x\right\} dx = \frac{1}{10}\sin(5x) + \frac{1}{2}\sin x + C$

(3) $\displaystyle\int \sin(4x)\sin x dx = \int \left\{\frac{1}{2}\cos(3x) - \frac{1}{2}\cos(5x)\right\} dx = \frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x) + C$

＜ 4 ページ. 不定積分の検証 ＞

問の解答

(1) $\left(\dfrac{1}{4}(x^4-1)^4\right)' = \dfrac{1}{4}\times 4(x^4-1)^3\times(4x^3) = 4x^3(x^4-1)^3$ より正しくない。

(2) $\left(\dfrac{1}{2}\log\left|x^2-1\right|\right)' = \dfrac{1}{2}\times\dfrac{(x^2-1)'}{x^2-1} = \dfrac{1}{2}\times\dfrac{2x}{x^2-1} = \dfrac{x}{x^2-1}$ より正しい。

(3) $(x^2e^x-2xe^x+2e^x)' = 2xe^x+x^2e^x-2e^x-2xe^x+2e^x = x^2e^x$ より正しい。

＜ 5ページ. 数列の類推 ＞

問1の解答

$a_1 = 1$, $a_2 = 5$, $a_3 = 14$, $a_4 = 30$, $a_5 = 55$,

$b_1 = 2$, $b_2 = 3$, $b_3 = 4$, $b_4 = 5$, $b_5 = 6$,

$b_n = n + 1$,

問2の解答

$a_1 = 1$, $a_2 = 3$, $a_3 = 6$, $a_4 = 10$, $a_5 = 15$,

$b_1 = 1$, $b_2 = 9$, $b_3 = 36$, $b_4 = 100$, $b_5 = 225$,

$b_n = \{a_n\}^2$,

＜ 6 ページ. 和の記号 $\sum$(シグマ) 1 ＞

問の解答

(1) $1 + 2 + 3 + 4 + 5 + 6 + 7$

(2) $1^4 + 2^4 + 3^4 + 4^4$

(3) $3 + 5 + 7 + 9 + 11$

(4) $-7 - 2 + 3 + 8 + 13 + 18$

(5) $1 + 1 + 1 + 1 + 1 + 1 + 1$

＜ 7 ページ. 和の記号 $\sum$(シグマ) 2 ＞

問 1 の解答

(1) $\displaystyle\sum_{k=1}^{n} k$ (2) $\displaystyle\sum_{k=1}^{n} (2k-1)\times 2k$ (3) $\displaystyle\sum_{k=1}^{6} (3k-2)$ (4) $\displaystyle\sum_{k=1}^{20} (5k)$

問 2 の解答

(1) $1+8+17+28+41$

(2) $10+26+48+76+100$

(3) $1+4+4^2+\cdots+4^n$

＜ 8ページ. 和の記号 $\sum$(シグマ) 3 ＞

問１の解答

(1) $2\sum_{k=1}^{n} k + 3\sum_{k=1}^{n} 1 = 2 \times \dfrac{n(n+1)}{2} + 3n = n^2 + 4n$

(2) $8\sum_{k=1}^{n} k - 5\sum_{k=1}^{n} 1 = 8 \times \dfrac{n(n+1)}{2} - 5n = 4n^2 - n$

問２の解答

(1) $\sum_{k=1}^{n} (2k-1) = 2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 = 2 \times \dfrac{n(n+1)}{2} - n = n^2$

(2) $\sum_{k=1}^{n} (5k-3) = 5\sum_{k=1}^{n} k - 3\sum_{k=1}^{n} 1 = 5 \times \dfrac{n(n+1)}{2} - 3n = \dfrac{5}{2}n^2 - \dfrac{1}{2}n$

(3) $\sum_{k=1}^{n} (7k-4) = 7\sum_{k=1}^{n} k - 4\sum_{k=1}^{n} 1 = 7 \times \dfrac{n(n+1)}{2} - 4n = \dfrac{7}{2}n^2 - \dfrac{1}{2}n$

＜ 9 ページ. 和の記号 $\sum$(シグマ) 4 ＞

問 1 の解答

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

問 2 の解答

(1) $\displaystyle\sum_{k=1}^{7} k^2 = \frac{7 \times 8 \times (2 \times 7 + 1)}{6} = \frac{7 \times 8 \times 15}{6} = 140$

(2) $\displaystyle\sum_{k=1}^{n+1} k^2 = \frac{(n+1)\big((n+1)+1\big)\big(2(n+1)+1\big)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$

< 10 ページ. 和の記号 $\sum$(シグマ) 5 >

問 1 の解答

$$\sum_{k=1}^{n} k^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

問 2 の解答

$$(1)\sum_{k=1}^{7} k^3 = \left\{\frac{7\times 8}{2}\right\}^2 = 28^2 = 784$$

$$(2)\sum_{k=1}^{n-1} k^3 = \left\{\frac{(n-1)n}{2}\right\}^2$$

＜ 11 ページ. 和の記号 $\sum$(シグマ) 6 ＞

問 1 の解答

(1) $x_2 + x_3 + x_4$

(2) $y_3 + y_4 + y_5 + y_6$

(3) $1^2 + 2^2 + 3^2 + \cdots + n^2$

(4) $2^3 + 3^3 + 4^3 + \cdots + n^3$

問 2 の解答

$$\sum_{i=2}^{4}\{\sum_{j=4}^{5}(x_i \times y_i)\} \;=\; \sum_{i=2}^{4} 2x_iy_i \;=\; 2x_2y_2 + 2x_3y_3 + 2x_4y_4$$

＜ 13 ページ. 区分求積法 2 ＞

問 1 の解答

$$\sum_{k=1}^{n-1} k^2 = \boxed{\frac{(n-1)n(2n-1)}{6}}$$

$$S_n = \left\{\frac{1}{6}(n-1)n(2n-1)\right\}\left(\frac{1}{n}\right)^3 = \frac{1}{6}\left(1-\boxed{\frac{1}{n}}\right)\left(2-\boxed{\frac{1}{n}}\right)$$

問 2 の解答

$$S_1 = 0\,, \qquad S_2 = \frac{1}{6}\times\frac{1}{2}\times\frac{3}{2} = \frac{1}{8}\,, \qquad S_3 = \frac{1}{6}\times\frac{2}{3}\times\frac{5}{3} = \frac{5}{27}$$

問 3 の解答

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\frac{1}{6}\times\left(1-\frac{1}{n}\right)\times\left(2-\frac{1}{n}\right) = \frac{1}{6}\times 1\times 2 = \frac{1}{3}$$

< 14ページ. 区分求積法 3 >

問の解答

(1) $S_n^* = x_1^2 h + x_2^2 h + x_3^2 h + \cdots + x_n^2 h$

(2)
$$\begin{aligned} S_n^* &= (h)^2 h + (2h)^2 h + (3h)^2 h + \cdots + (nh)^2 h \\ &= (1^2 + 2^2 + 3^2 + \cdots + n^2) h^3 \\ &= \left(\sum_{k=1}^{n} k^2 \right) \times h^3 \end{aligned}$$

(3) $S_n^* = \left(\dfrac{n(n+1)(2n+1)}{6} \right) \times \left(\dfrac{1}{n} \right)^3 = \dfrac{1}{6} \left(1 + \dfrac{1}{n} \right) \left(2 + \dfrac{1}{n} \right)$

(4) $S_1^* = \dfrac{1}{6} \times 2 \times 3 = 1$, $S_2^* = \dfrac{1}{6} \times \dfrac{3}{2} \times \dfrac{5}{2} = \dfrac{5}{8}$, $S_3^* = \dfrac{1}{6} \times \dfrac{4}{3} \times \dfrac{7}{3} = \dfrac{14}{27}$

(5) $\displaystyle\lim_{n\to\infty} S_n^* = \lim_{n\to\infty} \frac{1}{6} \times \left(1 + \frac{1}{n} \right) \times \left(2 + \frac{1}{n} \right) = \frac{1}{6} \times 1 \times 2 = \frac{1}{3}$

(6) $S = \dfrac{1}{3}$

< 15 ページ. 区分求積法 4 >

問 1 の解答

(1) $$\begin{aligned} S_n^* &= h^3h + (2h^3)h + \cdots + \big((n-1)h\big)^3 h \\ &= \{1^3 + 2^3 + \cdots + (n-1)^3\}h^4 \\ &= \left\{\sum_{k=1}^{n-1} k^3\right\} h^4 \end{aligned}$$

(2) $$S_n = \left\{\frac{(n-1)n}{2}\right\}^2 \times \left(\frac{1}{n}\right)^4 = \frac{1}{4}\left(1-\frac{1}{n}\right)^2$$

(3) $$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{4}\left(1-\frac{1}{n}\right)^2 = \frac{1}{4}$$

問 2 の解答

$$S_n^* = \left\{\sum_{k=1}^{n-1} k^3\right\} h^4 = \left\{\frac{(n+1)n}{2}\right\}^2 \times \left(\frac{1}{n}\right)^4 = \frac{1}{4}\left(1+\frac{1}{n}\right)^2$$

$$\lim_{n\to\infty} S_n^* = \lim_{n\to\infty} \frac{1}{4}\left(1+\frac{1}{n}\right)^2 = \frac{1}{4}$$

問 3 の解答

$$S = \frac{1}{4}$$

＜ 16 ページ. 区分求積法 5 ＞

問 1 の解答

(1) $S_n(x) = {x_1}^2\,h + {x_2}^2\,h + \cdots + {x_{n-1}}^2\,h$

(2)
$$\begin{aligned} S_n(x) &= h^2h + (2h)^2h + \cdots + \big((n-1)h\big)^2h \\ &= \{1^2 + 2^2 + \cdots + (n-1)^2\}h^3 \\ &= \left\{\sum_{k=1}^{n-1} k^2\right\} h^3 \end{aligned}$$

(3) $S_n(x) = \dfrac{(n-1)n(2n-1)}{6} \times \left(\dfrac{x}{n}\right)^3 = \dfrac{1}{6}\left(1-\dfrac{1}{n}\right)\left(2-\dfrac{1}{n}\right)x^3$

(4) $\displaystyle\lim_{n\to\infty} S_n(x) = \lim_{n\to\infty} \frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)x^3 = \frac{1}{3}x^3$

問 2 の解答

(1)
$$\begin{aligned} S_n^*(x) &= {x_1}^2\,h + {x_2}^2\,h + \cdots + {x_n}^2\,h \\ &= \left\{\sum_{k=1}^{n} k^2\right\} h^3 = \frac{n(n+1)(2n+1)}{6} \times \left(\frac{x}{n}\right)^3 = \frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)x^3 \end{aligned}$$

(2) $\displaystyle\lim_{n\to\infty} S_n^*(x) = \lim_{n\to\infty} \frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)x^3 = \frac{1}{3}x^3$

問 3 の解答

$S(x) = \dfrac{1}{3}x^3$

< 17 ページ. 区分求積法 6 >

問 1 の解答

(1) $S_n(x) = {x_1}^3\, h + {x_2}^3\, h + \cdots + {x_{n-1}}^3\, h$

(2)
$$\begin{aligned} S_n(x) &= h^3 h + (2h^3)h + \cdots + \big((n-1)h\big)^3 h \\ &= \big(1^3 + 2^3 + \cdots + (n-1)^3\big)h^4 \\ &= \left\{\sum_{k=1}^{n-1} k^3\right\} h^4 \end{aligned}$$

(3) $S_n(x) = \left\{\dfrac{(n-1)n}{2}\right\}^2 \left(\dfrac{x}{n}\right)^4 = \dfrac{1}{4}\left(1-\dfrac{1}{n}\right)^2 x^4$

(4) $\displaystyle\lim_{n\to\infty} S_n(x) = \lim_{n\to\infty} \frac{1}{4}\left(1-\frac{1}{n}\right)^2 x^4 = \frac{1}{4}x^4$

問 2 の解答

(1) $S_n^*(x) = \left(\displaystyle\sum_{k=1}^{n} k^3\right)\left(\dfrac{x}{n}\right)^4 = \dfrac{1}{4}\left(1+\dfrac{1}{n}\right)^2 x^4$

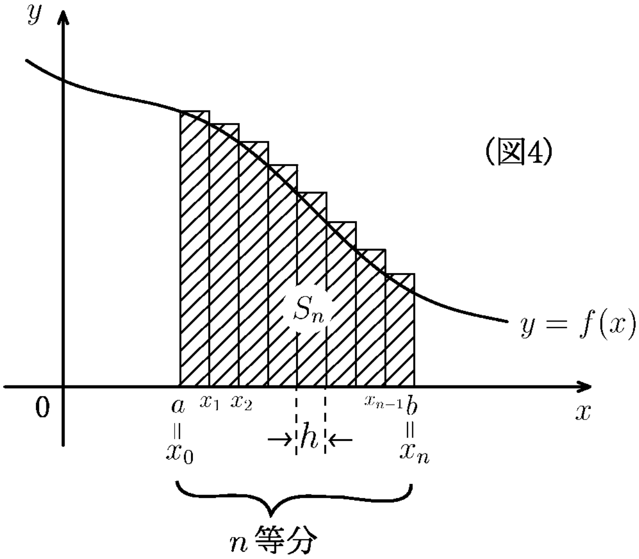
(2) $\displaystyle\lim_{n\to\infty} S_n^*(x) = \lim_{n\to\infty} \frac{1}{4}\left(1+\frac{1}{n}\right)^2 x^4 = \frac{1}{4}x^4$

問 3 の解答

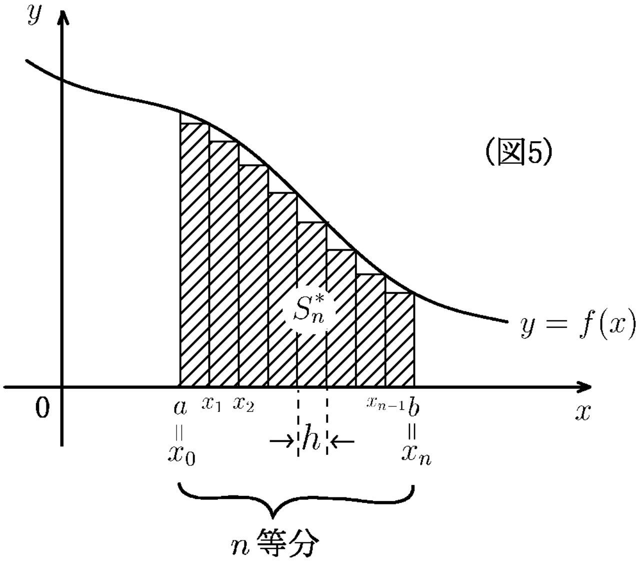
$S(x) = \dfrac{1}{4}x^4$

＜ 18 ページ. 定積分の定義 ＞

問の解答



(図4)



(図5)

＜ 21 ページ. 定積分の性質 ＞

問 1 の解答

$$\int_0^1 \left\{-x^3\right\} dx = -\int_0^1 x^3 dx = -\frac{1}{4}$$

問 2 の解答

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx = S_1 - S_2$$

＜ 22 ページ. 面積関数 $S(x)$ ＞

問 1 の解答

(1) $S(x) = x$

(2) $S(x) = \dfrac{1}{2}x^2$

問 2 の解答

(1) $S(x) = \dfrac{1}{3}x^3$

(2) $S(x) = \dfrac{1}{4}x^4$

問 3 の解答

(1) $S(x) = \dfrac{1}{5}x^5$

(2) $S(x) = \dfrac{1}{n+1}x^{n+1}$

問 4 の解答

$\Big(S(x)\Big)' = f(x)$ $\Big(S(x)$ の導関数は $f(x)\Big)$

または

$$\int f(x)dx = S(x) + C$$

< 24 ページ. 微分積分学の基本定理 1 >

問の解答

$$
\begin{aligned}
\frac{1}{h}\left\{\int_a^{x+h} f(x)dx - \int_a^{x} f(x)dx\right\} &= \frac{1}{-\delta}\left\{\int_a^{x-\delta} f(x)dx - \int_a^{x} f(x)dx\right\} \\
&= \frac{1}{\delta}\int_{x-\delta}^{x} f(x)dx = \frac{1}{h}\int_x^{\boxed{\;x+h\;}} f(x)dx
\end{aligned}
$$

< 25 ページ. 微分積分学の基本定理 2 >

問の解答

[定理 8]

<証明> 定理 5 と定理 7 より

$$S'(x) = \lim_{h \to 0} \frac{S(x+h) - S(x)}{h} = \lim_{h \to 0} \frac{1}{h} \left\{ \int_a^{x+h} f(x)dx - \int_a^x f(x)dx \right\}$$

$$= \lim_{h \to 0} \frac{1}{h} \int_{\boxed{x}}^{x+h} f(x)dx = f(x)$$

(証明終)

[定理 9]

<証明> $S(x) = \displaystyle\int_a^x f(x)dx$ とおくと定理 8 より $S'(x) = f(x)$ だから

$$\big(F(x) - S(x)\big)' = F'(x) - S'(x) = f(x) - f(x) = 0$$

微分して 0(ゼロ) になる関数 (⇔ 傾きが常に 0(ゼロ)) は定数だから

$$F(x) - S(x) = C \qquad (C \text{ は定数})$$

とおける。従って

$$F(x) = S(x) + C = \int_a^{\boxed{x}} f(x)dx + C$$

より

$$F(b) - F(a) = \{S(b) + C\} - \{S(a) + C\} = S(b) - S(a)$$

$$= \int_a^{\boxed{b}} f(x)dx - \int_a^{\boxed{a}} f(x)dx = \int_a^b f(x)dx$$

(証明終)

< 26 ページ. 定積分 1 >

問の解答

(1) $[x]_4^7 = 7 - 4 = 3$

(2) $\left[\frac{1}{2}x^2\right]_{-1}^{3} = \frac{1}{2} \times 3^2 - \frac{1}{2} \times (-1)^2 = 4$

(3) $\left[\frac{1}{3}x^3\right]_{-2}^{1} = \frac{1}{3} \times 1^3 - \frac{1}{3} \times (-2)^3 = \frac{1+8}{3} = 3$

(4) $\left[\frac{1}{4}x^4\right]_{-2}^{2} = \frac{1}{4} \times 2^4 - \frac{1}{4}(-2)^4 = 0$

(5) $\left[\frac{1}{5}x^5\right]_{-1}^{2} = \frac{1}{5} \times 2^5 - \frac{1}{5}(-1)^5 = \frac{32+1}{5} = \frac{33}{5}$

＜ 27 ページ. 定積分 2 ＞

問 1 の解答

(1) $\displaystyle\int_a^b dx = \int_a^b 1dx = \Big[x\Big]_a^b = b - a$

(2) $\displaystyle\int_a^b x^n dx = \left[\frac{1}{n+1}x^{n+1}\right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$

(3) $\displaystyle\int_a^b \frac{1}{x}dx = \Big[\log x\Big]_a^b = \log\left(\frac{b}{a}\right)$

(4) $\displaystyle\int_a^b e^x dx = \Big[e^x\Big]_a^b = e^b - e^a$

(5) $\displaystyle\int_a^b \cos x dx = \Big[\sin x\Big]_a^b = \sin b - \sin a$

(6) $\displaystyle\int_a^b \sin x dx = \Big[-\cos x\Big]_a^b = -\cos b + \cos a$

問 2 の解答

(1) $\displaystyle\int_4^{10} dx = \Big[x\Big]_4^{10} = 10 - 4 = 6$

(2) $\displaystyle\int_{-1}^1 (x^2 + x^3 + x^4)dx = \left[\frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5\right]_{-1}^1 = \frac{16}{15}$

(3) $\displaystyle\int_1^5 \frac{1}{x^2}dx = \left[-\frac{1}{x}\right]_1^5 = \frac{4}{5}$

(4) $\displaystyle\int_1^2 \frac{1}{x^3}dx = \left[-\frac{1}{2x^2}\right]_1^2 = \frac{3}{8}$

(5) $\displaystyle\int_4^9 \sqrt{x}dx = \left[\frac{2}{3}x\sqrt{x}\right]_4^9 = \frac{38}{3}$

(6) $\displaystyle\int_1^8 \sqrt[3]{x}dx = \left[\frac{3}{4}x\sqrt[3]{x}\right]_1^8 = \frac{45}{4}$

(7) $\displaystyle\int_0^9 \frac{1}{\sqrt{x}}dx = \Big[2\sqrt{x}\Big]_0^9 = 6$

(8) $\displaystyle\int_1^{e^2} \frac{1}{x}dx = \Big[\log x\Big]_1^{e^2} = 2$

(9) $\displaystyle\int_2^4 \frac{3}{x}dx = \Big[3\log x\Big]_2^4 = 3\log 2$

(10) $\displaystyle\int_0^2 e^x dx = \Big[e^x\Big]_0^2 = e^2 - 1$

(11) $\displaystyle\int_{-1}^1 4e^x dx = \Big[4e^x\Big]_{-1}^1 = 4e - \frac{4}{e}$

(12) $\displaystyle\int_0^\pi \sin x dx = \Big[-\cos x\Big]_0^\pi = 2$

(13) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = \Big[\sin x\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$

(14) $\displaystyle\int_0^{\frac{\pi}{2}} 3\sin x dx = \Big[-3\cos x\Big]_0^{\frac{\pi}{2}} = 3$

＜ 28 ページ. 定積分 3 ＞

問の解答

(1) $2\int_0^1 x^4 dx = 2\left[\frac{1}{5}x^5\right]_0^1 = \frac{2}{5}$

(2) $2\int_0^1 x^6 dx = 2\left[\frac{1}{7}x^7\right]_0^1 = \frac{2}{7}$

＜ 29 ページ. 定積分の積分変数 ＞

問の解答

(1) $\left[4t - 4.9t^2\right]_1^3 = 12 - 4.9 \times 9 - (4 - 4.9) = -31.2$

(2) $\left[\pi r^2\right]_0^R = \pi R^2$

(3) $[-\cos\theta]_0^\pi = 2$

(4) $\left[\dfrac{1}{n+1}u^{n+1}\right]_a^b = \dfrac{b^{n+1}}{n+1} - \dfrac{a^{n+1}}{n+1}$

(5) $\left[\dfrac{2}{3}u\sqrt{u}\right]_1^9 = \dfrac{2}{3}(27-1) = \dfrac{52}{3}$

< 30 ページ. 定積分の置換積分法 1 >

問の解答

(1) $u = x^3 + 1$ とおくと $\displaystyle\int_{-1}^{1} 3x^2(x^3+1)dx = \int_0^2 u^4 du = \left[\frac{1}{5}u^5\right]_0^2 = \frac{32}{5}$

(2) $u = x^2+1$ とおくと $\displaystyle\int_0^2 2x\sqrt{x^2+1}dx = \int_1^5 \sqrt{u}du = \left[\frac{2}{3}u\sqrt{u}\right]_1^5 = \frac{2}{3}(5\sqrt{5}-1)$

(3) $u = x^4 + 1$ とおくと $\displaystyle\int_0^1 \frac{4x^3}{(x^4+1)^2}dx = \int_1^2 \frac{1}{u^2}du = \left[-\frac{1}{u}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

＜ 31 ページ. 定積分の置換積分法 2 ＞

問の解答

(1) $u = x^2 + 2$ とおくと $\dfrac{du}{dx} = 2x$ より、

$$\int_0^1 x(x^2+2)^3 dx = \int_2^3 u^3 \times \frac{1}{2} du = \left[\frac{1}{8}u^4\right]_2^3 = \frac{1}{8}(81-16) = \frac{65}{8}$$

(2) $u = x^2$ とおくと、

$$\int_0^3 xe^{x^2} dx = \int_0^9 e^u \times \frac{1}{2} du = \left[\frac{1}{2}e^u\right]_0^9 = \frac{1}{2}e^9 - \frac{1}{2}$$

(3) $u = x^3 + 2$ とおくと、

$$\int_{-1}^2 \frac{x^2}{x^3+2} dx = \int_1^{10} \frac{1}{u} \times \frac{1}{3} du = \left[\frac{1}{3}\log u\right]_1^{10} = \frac{1}{3}\log 10$$

(4) $u = x^2 + 1$ とおくと、

$$\int_0^2 \frac{x}{x^2+1} dx = \int_1^5 \frac{1}{u^3} \times \frac{1}{2} du = \left[-\frac{1}{4u^2}\right]_1^5 = -\frac{1}{4}\left(\frac{1}{25} - 1\right) = \frac{6}{25}$$

< 32 ページ. 定積分の部分積分法 >

問の解答

(1) $$\int_{-1}^{1}(x+1)(x-1)^3dx = \int_{-1}^{1}(x+1)\times\left(\frac{(x-1)^4}{4}\right)'dx$$

$$= \left[(x+1)\times\frac{(x-1)^4}{4}\right]_{-1}^{1} - \int_{-1}^{1}\frac{(x-1)^4}{4}dx$$

$$= \left[-\frac{(x-1)^5}{20}\right]_{-1}^{1} = -0 + \frac{(-2)^5}{20} = -\frac{32}{20} = -\frac{8}{5}$$

(2) $$\int_{0}^{\frac{\pi}{2}} x\sin x dx = \int_{0}^{\frac{\pi}{2}} x\times(-\cos x)'dx$$

$$= \Big[-x\cos x\Big]_{0}^{\frac{\pi}{2}} - \int_{0}^{\frac{\pi}{2}}(-\cos x)dx = \Big[\sin x\Big]_{0}^{\frac{\pi}{2}} = 1$$

(3) $$\int_{0}^{1} xe^x dx = \int_{0}^{1} x\big(e^x\big)'dx = \big[xe^x\big]_0^1 - \int_0^1 e^x dx$$

$$= e - \big[e^x\big]_0^1 = e - (e-1) = 1$$

＜ 33ページ. 面積 1 ＞

問の解答

$$S = \int_{-1}^{2} (-x^2 + 2x + 4)dx - \int_{-1}^{2} x^2 dx$$

$$= \int_{-1}^{2} (-2x^2 + 2x + 4)dx$$

$$= \left[-\frac{2}{3}x^3 + x^2 + 4x \right]_{-1}^{2}$$

$$= \left(-\frac{16}{3} + 4 + 8 \right) - \left(\frac{2}{3} + 1 - 4 \right)$$

$$= -\frac{18}{3} + 12 + 3 = 9$$

＜ 34 ページ. 面積 2 ＞

問 1 の解答

$$S = \int_a^b \{f(x) + C\}\,dx - \int_a^b \{g(x) + C\}\,dx$$

$$= \int_a^b \{f(x) - g(x)\}\,dx$$

問 2 の解答

$$S = \int_{-1}^{1} \left\{(-x^2 + 2x + 1) - (x^2 + 2x - 1)\right\} dx$$

$$= \int_{-1}^{1} \left\{-2x^2 + 2\right\} dx$$

$$= \left[-\frac{2}{3}x^3 + 2x\right]_{-1}^{1}$$

$$= \left(-\frac{2}{3} + 2\right) - \left(\frac{2}{3} - 2\right) = 4 - \frac{4}{3} = \frac{8}{3}$$

< 35 ページ. 円の面積 >

問の解答

(1) $x = r\sin u$

$$\frac{dx}{du} = r\cos u$$

$$\int_0^r \sqrt{r^2 - x^2}dx = \int_{u=0}^{u=\frac{\pi}{2}} \sqrt{r^2 - r^2\sin^2 u}\ r\cos u du$$

$$= \int_{u=0}^{u=\frac{\pi}{2}} r^2\cos^2 u du = \int_0^{\frac{\pi}{2}} \frac{r^2}{2}\left(1 + \cos(2u)\right) du$$

$$= \left[\frac{r^2}{2}\left(u + \frac{1}{2}\sin(2u)\right)\right]_0^{\frac{\pi}{2}} = \frac{r^2}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}r^2$$

(2) (1) より、$\dfrac{S}{4} = \dfrac{\pi}{4}r^2$

$$\Downarrow$$

(答) $S = \pi r^2$

＜ 36 ページ. 体積 1 ＞

問 1 の解答

$$v_k = f(x_k)h = \frac{25}{98}(x_k)^2 h = \frac{25}{98} \times (kh)^2 h = \frac{25}{98}k^2 \left(\frac{7}{n}\right)^3 = \frac{175k^2}{2n^3}$$

問 2 の解答

$$V_n = \sum_{k=1}^{n-1} \frac{175}{2n^3}k^2 = \frac{175}{2n^3} \times \frac{(n-1)n(2n-1)}{6} = \frac{175}{12}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

問 3 の解答

$$\lim_{n\to\infty} V_n = \lim_{n\to\infty} \frac{175}{12}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) = \frac{175}{12} \times 1 \times 2 = \frac{175}{6}$$

問 4 の解答

$$v_k = f(x_k)h = \frac{175k^2}{2n^3}$$

問 5 の解答

$$V_n^* = \sum_{k=1}^{n} \frac{175}{2n^3}k^2 = \frac{175}{2n^3} \times \frac{n(n+1)(2n+1)}{6} = \frac{175}{12}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

問 6 の解答

$$\lim_{n\to\infty} V_n^* = \lim_{n\to\infty} \frac{175}{12}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) = \frac{175}{6}$$

問 7 の解答

$$V = \frac{175}{6}$$

＜ 37 ページ. 体積 2 ＞

問 1 の解答

$$V = \int_0^7 \frac{25}{98} x^2 dx = \left[\frac{25}{98} \times \frac{x^3}{3} \right]_0^7 = \frac{25}{98} \times \frac{7^3}{3} = \frac{175}{6}$$

問 2 の解答

(1) $f(x) = \frac{1}{2} \times \left(\frac{x}{2} \right) \times \left(\frac{2}{3} x \right) = \frac{x^2}{6}$

(2) $V = \int_0^6 f(x) dx = \int_0^6 \frac{x^2}{6} dx = \left[\frac{x^3}{18} \right]_0^6 = \frac{6^3}{18} = 12$

＜ 38 ページ. 体積 3 ＞

問の解答

(1) $\mathrm{A}_1'\mathrm{C}_1' = \dfrac{5}{7}x$

(2) $\mathrm{A}_2'\mathrm{C}_2' = \dfrac{5}{7}x$

(3) $S(x) = \dfrac{1}{2} \times \left(\dfrac{5}{7}x\right)^2 = \dfrac{25}{98}x^2$

(4) $\displaystyle\int_0^7 S(x)dx = \int_0^7 \frac{25}{98}x^2dx = \left[\frac{25}{98} \times \frac{x^3}{3}\right]_0^7 = \frac{25 \times 7^3}{98 \times 3} = \frac{175}{6}$

< 39 ページ. 体積 4 >

問の解答

$$S(x) = \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$$

$$V = \int_0^4 S(x)dx = \int_0^4 \frac{x^2}{4}dx = \left[\frac{1}{12}x^3\right] = \frac{4^3}{12} = \frac{16}{3}$$

＜ 40 ページ. 体積 5 ＞

問 1 の解答

(1) $S(x)=\pi\left\{f(x)\right\}^2$

(2) $V=\int_a^b S(x)dx=\int_a^b \pi\left\{f(x)\right\}^2 dx$

問 2 の解答

$$V=\int_0^h \pi\left(\frac{r}{h}x\right)^2 dx=\int_0^h \frac{\pi r^2}{h^2}x^2 dx=\left[\frac{\pi r^2}{h^2}\times\frac{x^3}{3}\right]_0^h=\frac{\pi r^2}{h^2}\times\frac{h^3}{3}=\frac{\pi r^2 h}{3}$$

問 3 の解答

$$\begin{aligned}
V&=\int_{-r}^{r}\pi\left\{\sqrt{r^2-x^2}\right\}^2 dx\\
&=\int_{-r}^{r}\pi\left(r^2-x^2\right)dx\\
&=\pi\left[r^2x-\frac{x^3}{3}\right]_{-r}^{r}\\
&=\pi\left\{\left(r^3-\frac{r^3}{3}\right)-\left(-r^3+\frac{r^3}{3}\right)\right\}\\
&=\frac{4\pi}{3}r^3
\end{aligned}$$