高知工科大学

基礎数学ワークブック

(2002年度版)

Series **A**

No. **8**

解答

＜１ページ. 複素数の四則演算 １＞

問１の解答

(1) $(2+i)+(3-i)$

$= 5$

(2) $(4-i)-(5-3i)$

$= -1+2i$

(3) $\left(0.13+\dfrac{1}{2}i\right)+\left(\dfrac{3}{4}-1.5i\right)$

$= 0.13+0.75+\left(\dfrac{1}{2}-\dfrac{3}{2}\right)i = 0.88-i$

(4) $\left(\dfrac{1}{4}-\dfrac{1}{3}i\right)-\left(\dfrac{1}{8}-\dfrac{1}{3}i\right)$

$= \dfrac{1}{8}$

(5) $\left(\sqrt{3}-i\right)+\left(\sqrt{1}-2i\right)$

$= \sqrt{3}+1-3i$

(6) $\left(\dfrac{1}{4}-\sqrt{2}i\right)-\left(\dfrac{1}{3}+\sqrt{3}i\right)$

$= -\dfrac{1}{12}-(\sqrt{2}+\sqrt{3})i$

問２の解答

(1) $3(4+i)$

$= 12+3i$

(2) $6\left(\dfrac{1}{4}-\dfrac{1}{2}i\right)$

$= \dfrac{3}{2}-3i$

(3) $3(6-2i)-4(2-i)$

$= 18-6i-8+4i$

$= 10-2i$

(4) $\sqrt{3}\left(\dfrac{1}{\sqrt{3}}-\sqrt{3}i\right)+\left(\dfrac{1}{3}-2i\right)$

$= 1-3i+\dfrac{1}{3}-2i$

$= \dfrac{4}{3}-5i$

< 2 ページ. 複素数の四則演算 2 >

問の解答

(1) $i^3 = -i$　(2) $i^4 = 1$　(3) $i^5 = i$

(4) $i^6 = -1$　(5) $i^7 = -i$　(6) $i^8 = 1$

(7) $(1+i)(1-i) = 1-i^2 = 2$

(8) $(2+\sqrt{3}i)(2-\sqrt{3}i) = 4-3i^2 = 7$

(9) $$\left(\frac{\sqrt{3}+i}{2}\right)\left(\frac{\sqrt{3}-i}{2}\right) = \frac{3-i^2}{4} = 1$$

(10) $(-1+i)^2 = 1-2i+i^2 = -2i$

(11) $(-1-i)^2 = 1+2i+i^2 = 2i$

(12) $(4+2i)(2-3i) = 8-12i+4i-6i^2$
$= 14-8i$

(13) $(3-2i)(1-3i) = 3-9i-2i+6i^2$
$= -3-11i$

(14) $(3-i)^3 = 3^3 - 3\times 3^2 i + 3\times 3\times i^2 - i^3$
$= 27-27i-9-(-i)$
$= 18-26i$

＜ 3 ページ. 複素数の四則演算 3 ＞

問の解答

(1) $\dfrac{-1}{1+i} = \dfrac{-1(1-i)}{1^2-i^2} = \dfrac{i-1}{2}$

(2) $\dfrac{-1}{1-i} = \dfrac{-(1+i)}{1^2-i^2} = \dfrac{-1-i}{2}$

(3) $\dfrac{-i}{1-i} = \dfrac{-i(1+i)}{1^2-i^2} = \dfrac{-i+1}{2}$

$= \dfrac{1-i}{2}$

(4) $\dfrac{3}{\sqrt{5}-i} = \dfrac{3(\sqrt{5}+i)}{5-i^2} = \dfrac{3(\sqrt{5}+i)}{6}$

$= \dfrac{\sqrt{5}+i}{2}$

(5) $\dfrac{7}{3+\sqrt{5}i} = \dfrac{7(3-\sqrt{5}i)}{3^2-5i^2}$

$= \dfrac{7(3-\sqrt{5}i)}{14} = \dfrac{3-\sqrt{5}i}{2}$

(6) $\dfrac{-i}{1+i} = \dfrac{-i(1-i)}{1^2-i^2} = \dfrac{-i+i^2}{2}$

$= \dfrac{-1-i}{2}$

(7) $\dfrac{1}{\sqrt{3}i(\sqrt{3}+i)} = \dfrac{1}{3i-\sqrt{3}}$

$= \dfrac{3i+\sqrt{3}}{(3i)^2-3} = \dfrac{3i+\sqrt{3}}{-9-3} = -\dfrac{\sqrt{3}+3i}{12}$

(8) $\dfrac{\sqrt{2}}{\sqrt{2}-i} = \dfrac{\sqrt{2}(\sqrt{2}+i)}{2-i^2} = \dfrac{2+\sqrt{2}i}{3}$

(9) $\dfrac{1}{(\sqrt{2}-i)^2} = \dfrac{1}{2-2\sqrt{2}i+i^2}$

$= \dfrac{1}{1-2\sqrt{2}i} = \dfrac{1+2\sqrt{2}i}{1^2-(2\sqrt{2}i)^2}$

$= \dfrac{1+2\sqrt{2}i}{9}$

(10) $\dfrac{i}{(1+i)^4} = \dfrac{i}{1+4i+6i^2+4i^3+i^4}$

$= \dfrac{i}{1+4i-6-4i+1}$

$= -\dfrac{i}{4}$

＜ 4 ページ. 負の数の平方根 ＞

問の解答

(1) $\sqrt{(-3)\times(-4)\times(-5)}$

$= \sqrt{-60} = \sqrt{60}i = 2\sqrt{15}i$

(2) $\sqrt{-3}\times\sqrt{-4}\times\sqrt{-5} = \sqrt{3}i\times 2i\times\sqrt{5}i$

$= -2\sqrt{15}i$

(3) $\dfrac{\sqrt{12}}{\sqrt{-4}} = \dfrac{2\sqrt{3}}{2i} = \dfrac{\sqrt{3}i}{i^2} = -\sqrt{3}i$

(4) $\sqrt{\dfrac{12}{-4}} = \sqrt{-3} = \sqrt{3}i$

＜ 5 ページ.2 次方程式 ＞

問の解答

(1) $x^2 + x + 2 = 0$ $\quad x = \dfrac{-1 \pm \sqrt{1-8}}{2} = \dfrac{-1 \pm \sqrt{7}i}{2}$

(2) $x^2 + 3x + 9 = 1$

$x^2 + 3x + 8 = 0$ $\quad x = \dfrac{-3 \pm \sqrt{9-32}}{2} = \dfrac{-3 \pm \sqrt{23}i}{2}$

(3) $3x^2 - 5x + 4 = 0$ $\quad x = \dfrac{5 \pm \sqrt{25-48}}{6} = \dfrac{5 \pm \sqrt{23}i}{6}$

< 6ページ.2次式の因数分解 >

問の解答

(1) $x^2-2x+5=(x-1)^2+4=(x-1-2i)(x-1+2i)$

(2) $-5x^2+4x-3=-5\left(x-\dfrac{2-\sqrt{11}i}{5}\right)\left(x-\dfrac{2+\sqrt{11}i}{5}\right)$

$=-5\left(x-\dfrac{2}{5}+\dfrac{\sqrt{11}}{5}i\right)\left(x-\dfrac{2}{5}-\dfrac{\sqrt{11}}{5}i\right)=-5\left(x-\dfrac{2}{5}+\dfrac{\sqrt{11}}{5}i\right)\left(x-\dfrac{2}{5}-\dfrac{\sqrt{11}}{5}i\right)$

(3) $3x^2-3x+3=3\left(x^2-x+1\right)=3\left(x-\dfrac{1+\sqrt{3}i}{2}\right)\left(x-\dfrac{1-\sqrt{3}i}{2}\right)$

$=3\left(x-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i\right)\left(x-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)$

＜ 7 ページ. 高次式の因数分解 ＞

問の解答

(1) $x^3-1=(x-1)(x^2+x+1)=(x-1)\left(x-\dfrac{-1+\sqrt{3}i}{2}\right)\left(x-\dfrac{-1-\sqrt{3}i}{2}\right)$
$=(x-1)\left(x+\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i\right)\left(x+\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)$

(2) $x^3+8=(x+2)(x^2-2x+4)=(x+2)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i)$

(3) $x^4-1=(x^2-1)(x^2+1)=(x-1)(x+1)(x-i)(x+i)$

＜ 8 ページ. 高次方程式 ＞

問の解答

(1) $x^3 - 1 = 0 \quad (x-1)\left(x - \dfrac{-1+\sqrt{3}i}{2}\right)\left(x - \dfrac{-1-\sqrt{3}i}{2}\right) = 0$

(答) $x = 1\ ,\ \dfrac{-1+\sqrt{3}i}{2}\ ,\ \dfrac{-1-\sqrt{3}i}{2}$

(2) $x^3 + 27 = 0 \quad (x+3)(x^2 - 3x + 9) = 0$

(答) $x = -3\ ,\ \dfrac{3 \pm 3\sqrt{3}i}{2}$

(3) $x^4 - 1 = 0 \quad (x-1)(x+1)(x-i)(x+i) = 0$

(答) $x = \pm 1\ ,\ \pm i$

＜ 9 ページ. 共役複素数 ＞

問 1 の解答

(1) $z = 1$, $\overline{z} = 1$

(2) $z = i$, $\overline{z} = -i$

(3) $z = 1 - i$, $\overline{z} = 1 + i$

(4) $z = \dfrac{1+i}{2}$, $\overline{z} = \dfrac{1-i}{2}$

問 2 の解答

(1) $\dfrac{1}{2}(z + \overline{z})$

$= 4$

(2) $\dfrac{1}{2i}(z - \overline{z})$

$= \dfrac{1}{2i}\left(4 + 3i - (4 - 3i)\right)$

$= \dfrac{1}{2i} \times 6i = 3$

(3) $z\overline{z}$

$= 4^2 - 3^2 i^2 = 25$

問 3 の解答

(1) $\dfrac{1}{2}(z + \overline{z})$

$= a$

(2) $\dfrac{1}{2i}(z - \overline{z})$

$= b$

(3) $z\overline{z}$

$= a^2 + b^2$

＜ 10ページ. 絶対値 ＞

問 1 の解答

(1) $z = -1$ (2) $z = 7i$ (3) $z = 3 + 4i$ (4) $z = \dfrac{1+i}{2}$

$|z| = 1$ $|z| = 7$ $|z| = 5$ $|z| = \sqrt{\dfrac{1}{4} + \dfrac{1}{4}} = \dfrac{\sqrt{2}}{2}$

問 2 の解答

(1) $z = 4 - 3i$

$$|z|^2 = 4^2 + 3^2 = 25$$

$$\begin{aligned} z^2 &= (4-3i)^2 = 16 - 24i + 9i^2 \\ &= 7 - 24i \end{aligned}$$

$$|z^2| = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$$

(2) $z = 1 + i$

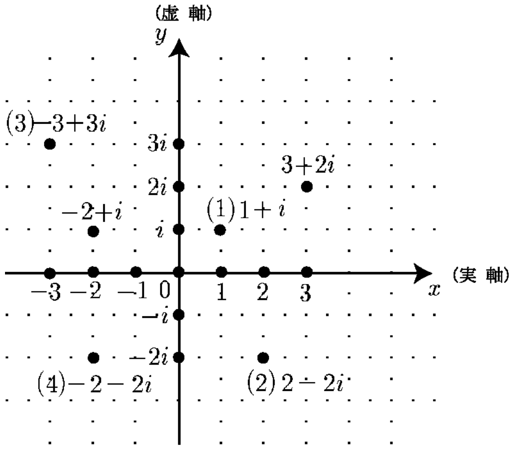
$$|z|^2 = 1^2 + 1^2 = 2$$

$$z^2 = (1+i)^2 = 2i$$

$$|z^2| = \sqrt{2^2} = 2$$

＜ 11 ページ. 複素平面 1 ＞

問 1 の解答



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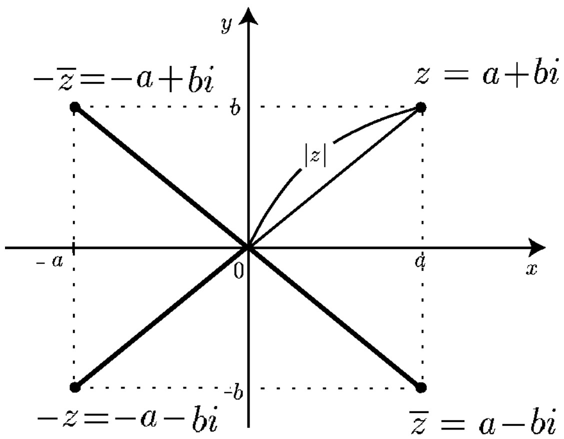


＜ 12 ページ. 複素平面 2 ＞

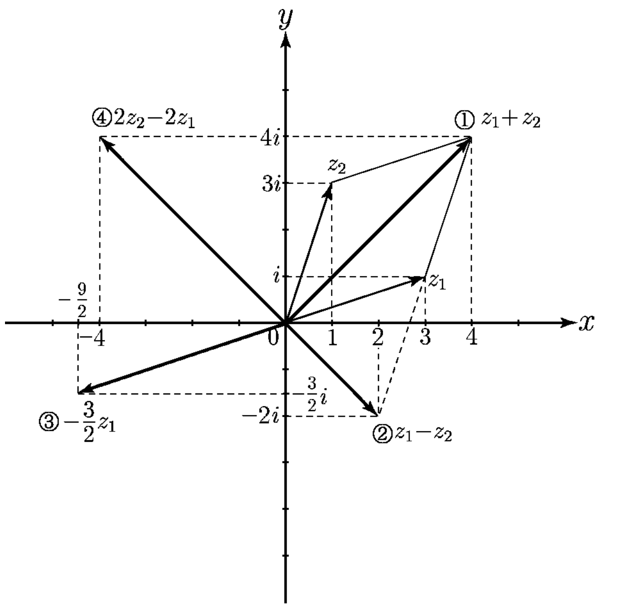
問の解答

① $z_1 + z_2 = (3+i) + (1+3i)$
$\qquad\qquad = 4 + 4i$

② $z_1 - z_2 = (3+i) - (1+3i)$
$\qquad\qquad = 2 - 2i$

③ $-\dfrac{3}{2}z_1 = -\dfrac{3}{2}(3+i)$
$\qquad\qquad = -\dfrac{9}{2} - \dfrac{3}{2}i$

④ $2z_2 - 2z_1 = 2(z_2 - z_1)$
$\qquad\qquad = 2\{(1+3i)-(3+i)\}$
$\qquad\qquad = 2(-2+2i)$
$\qquad\qquad = -4+4i$



$<$ 13 ページ. 複素数の i 倍 $>$

問の解答

(1) $z = 1 + i$

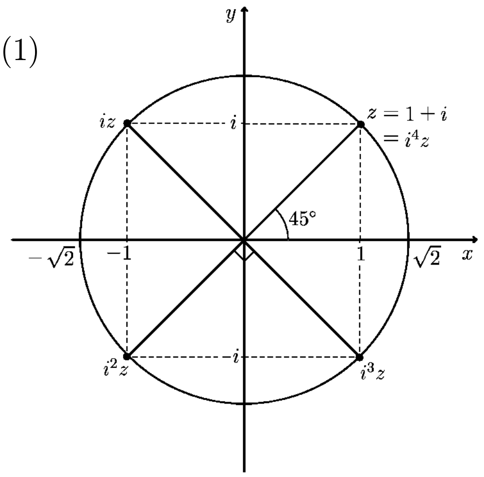
$$iz = (i + 1) = i - 1 = -1 + i$$

$$i^2 z = i(i - 1) = -1 - i$$

$$i^3 z = i(-1 - i) = -i + 1 = 1 - i$$

$$i^4 z = i(-i + 1) = 1 + i$$

(1)



(2) $z = 1 + \sqrt{3}i$

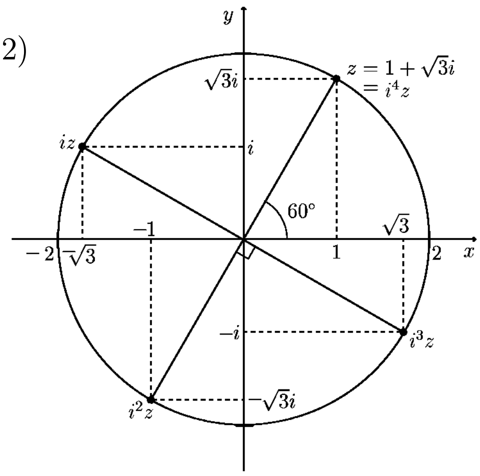
$$iz = i(1 + \sqrt{3}i) = i - \sqrt{3} = -\sqrt{3} + i$$

$$i^2 z = i(i - \sqrt{3}) = -1 - \sqrt{3}i$$

$$i^3 z = i(-1 - \sqrt{3}i) = -i + \sqrt{3} = \sqrt{3} - i$$

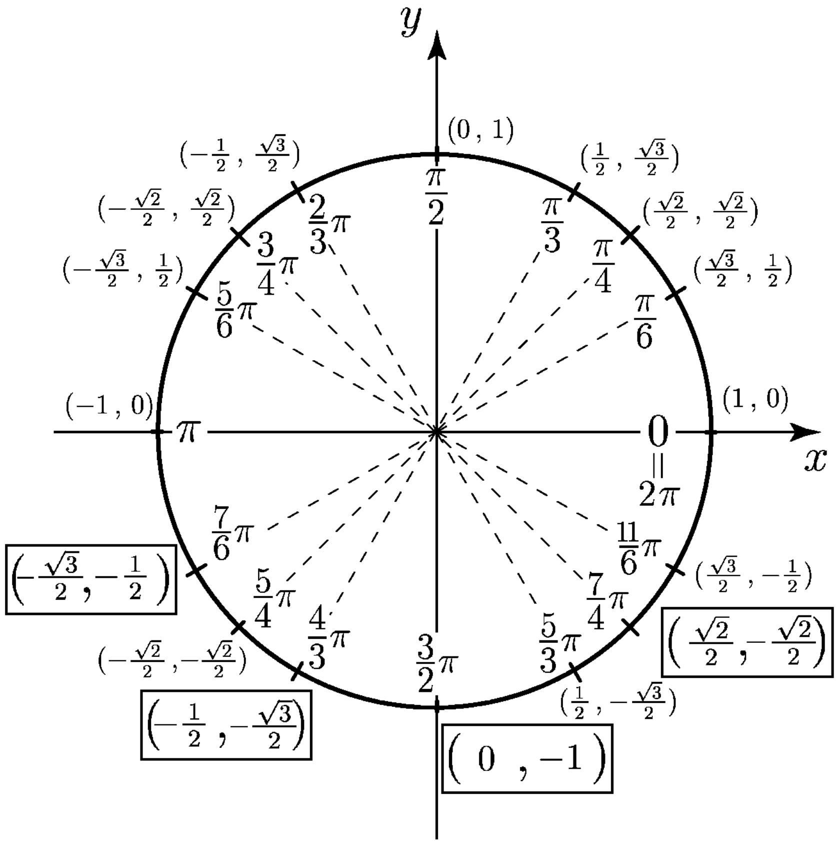
$$i^4 z = i(-i + \sqrt{3}) = 1 + \sqrt{3}i$$

(2)



＜ 14 ページ. 極座標 1 ＞

問 1 の解答



問 2 の解答

(1) $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right) = \left(\cos\dfrac{2}{3}\pi\ ,\ \sin\dfrac{2}{3}\pi\right)$

(2) $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right) = \left(\cos\dfrac{\pi}{4}\ ,\ \sin\dfrac{\pi}{4}\right)$

(3) $\left(1, 0\right) = (\cos 0\ ,\ \sin 0)$

(4) $\left(-\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right) = \left(\cos\dfrac{7}{6}\pi\ ,\ \sin\dfrac{7}{6}\pi\right)$

(5) $\left(0, -1\right) = \left(\cos\dfrac{3}{2}\pi\ ,\ \sin\dfrac{3}{2}\pi\right)$

(6) $\left(-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right) = \left(\cos\dfrac{5}{4}\pi\ ,\ \sin\dfrac{5}{4}\pi\right)$

＜ 15 ページ. 極座標表示 2 ＞

問の解答

(1) $\left(\ 3\ ,\ 3\ \right) = \left(\ 3\sqrt{2}\cos\dfrac{\pi}{4}\ ,\ 3\sqrt{2}\sin\dfrac{\pi}{4}\ \right)$

(2) $\left(1,\ -\sqrt{3}\right) = \left(2\cos\left(-\frac{\pi}{3}\right),\ 2\sin\left(-\frac{\pi}{3}\right)\right)$

$= \left(2\cos\left(\frac{5\pi}{3}\right),\ 2\sin\left(\frac{5\pi}{3}\right)\right)$

(3) $\left(\sqrt{3},\ 1\right) = \left(2\cos\dfrac{\pi}{6},\ 2\sin\dfrac{\pi}{6}\right)$

(4) $\left(\ -2\ ,\ -2\ \right) = \left(2\sqrt{2}\cos\left(\frac{5}{4}\pi\right),\ 2\sqrt{2}\sin\left(\frac{5}{4}\pi\right)\right)$

＜ 16 ページ. 絶対値 1 の複素数 ＞

問の解答

(1) $\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, (2) $\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)=\frac{1}{2}+\frac{\sqrt{3}}{2}i$, (3) $\cos\left(\frac{2}{3}\pi\right)+i\sin\left(\frac{2}{3}\pi\right)=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

(4) $\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, (5) $\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$, (6) $\cos(\pi)+i\sin(\pi)=-1$

(7) $\cos\left(\frac{7}{6}\pi\right)+i\sin\left(\frac{7}{6}\pi\right)=-\frac{\sqrt{3}}{2}-\frac{1}{2}i$, (8) $\cos\left(\frac{5}{4}\pi\right)+i\sin\left(\frac{5}{4}\pi\right)=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$, (9) $\cos\left(\frac{4}{3}\pi\right)+i\sin\left(\frac{4}{3}\pi\right)=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

(10) $\cos\left(\frac{3}{2}\pi\right)+i\sin\left(\frac{3}{2}\pi\right)=-i$, (11) $\cos\left(\frac{5}{3}\pi\right)+i\sin\left(\frac{5}{3}\pi\right)=\frac{1}{2}-\frac{\sqrt{3}}{2}i$, (12) $\cos\left(\frac{7}{4}\pi\right)+i\sin\left(\frac{7}{4}\pi\right)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

＜ 17 ページ. 極形式 1 ＞

問の解答

(1) $4i = 4\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

(2) $-2 = 2(\cos\pi + i\sin\pi)$

(3) $$-\sqrt{2}i = \sqrt{2}\left(\cos\left(\frac{3}{2}\pi\right) + i\sin\left(\frac{3}{2}\pi\right)\right)$$
$$\left(= \sqrt{2}\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)\right)$$

＜ 18 ページ. 極形式 2 ＞

問の解答

(1) $z = 1 + i = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

(2) $z = -1 - i = \sqrt{2}\left(\cos\left(\frac{5}{4}\pi\right) + i\sin\left(\frac{5}{4}\pi\right)\right) = \sqrt{2}\left(\cos\left(-\frac{3}{4}\pi\right) + i\sin\left(-\frac{3}{4}\pi\right)\right)$

(3) $z = 2\sqrt{2} + 2\sqrt{2}i = 4\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

(4) $z = -3 - \sqrt{3}i = 2\sqrt{3}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 2\sqrt{3}\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right)$

$= 2\sqrt{3}\left(\cos\left(-\frac{5}{6}\pi\right) + i\sin\left(-\frac{5}{6}\pi\right)\right)$

(5) $z = -\sqrt{18} + \sqrt{6}i = 2\sqrt{6}\left(-\sqrt{\frac{18}{24}} + \sqrt{\frac{6}{24}}i\right) = 2\sqrt{6}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= 2\sqrt{6}\left(\cos\left(\frac{5}{6}\pi\right) + i\sin\left(\frac{5}{6}\pi\right)\right)$

＜ 19 ページ. 複素数の積 ＞

問の解答

(1) $\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)z=\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)r(\cos\theta+i\sin\theta)$

$=r\left(\cos\left(\theta+\dfrac{\pi}{3}\right)+i\sin\left(\theta+\dfrac{\pi}{3}\right)\right)$

原点を中心として反時計まわりに $\dfrac{\pi}{3}(=60^\circ)$ 回転する

(2) $\left(\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}i\right)z=\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)r(\cos\theta+i\sin\theta)$

$=r\left(\cos\left(\theta+\dfrac{\pi}{4}\right)+i\sin\left(\theta+\dfrac{\pi}{4}\right)\right)$

原点を中心として反時計まわりに $\dfrac{\pi}{4}(=45^\circ)$ 回転する

(3) $iz=\left(\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}\right)r(\cos\theta+i\sin\theta)$

$=r\left(\cos\left(\theta+\dfrac{\pi}{2}\right)+i\sin\left(\theta+\dfrac{\pi}{2}\right)\right)$

原点を中心として反時計まわりに $\dfrac{\pi}{2}(=90^\circ)$ 回転する

＜ 20 ページ. 複素数の商 ＞

問の解答

(1) $$\frac{1+\sqrt{3}i}{\sqrt{3}+i} = \frac{2(\frac{1}{2}+\frac{\sqrt{3}}{2}i)}{2(\frac{\sqrt{3}}{2}+\frac{1}{2}i)} = \frac{\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)} = \cos\left(\frac{\pi}{3}-\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)$$

(2) $$\frac{1-i}{-1+i} = \frac{\sqrt{2}(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i)}{\sqrt{2}(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i)} = \frac{\cos\left(\frac{7}{4}\pi\right)+i\sin\left(\frac{7}{4}\pi\right)}{\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)} = \cos\pi + i\sin\pi$$

$$\Bigl(= \cos(-\pi)+i\sin(-\pi)\Bigr)$$

(3) $$\frac{1-i}{-\sqrt{3}+i} = \frac{\sqrt{2}(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i)}{2(-\frac{\sqrt{3}}{2}+\frac{1}{2}i)} = \frac{\sqrt{2}\left(\cos\left(\frac{7}{4}\pi\right)+i\sin\left(\frac{7}{4}\pi\right)\right)}{2\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)}$$

$$= \frac{\sqrt{2}}{2}\left(\cos\left(\frac{11}{12}\pi\right)+i\sin\left(\frac{11}{12}\pi\right)\right)$$

$$\left(= \frac{\sqrt{2}}{2}\left(\cos\left(-\frac{13}{12}\pi\right)+i\sin\left(-\frac{13}{12}\pi\right)\right)\right)$$

< 21 ページ. ド・モアブルの定理 >

問の解答

(1) $$(-\sqrt{3}+i)^3 = \left(2\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\right)^3 = 2^3\left(\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)\right)^3$$

$$= 8\left(\cos\left(\frac{5}{2}\pi\right)+i\sin\left(\frac{5}{2}\pi\right)\right) = 8i$$

(2) $$\left(\frac{-1+\sqrt{3}i}{2}\right)^6 = \left(\cos\left(\frac{2}{3}\pi\right)+i\sin\left(\frac{2}{3}\pi\right)\right)^6$$

$$= \cos(4\pi)+i\sin(4\pi) = 1$$

(3) $$\left(\frac{1-i}{2}\right)^4 = \left(\frac{1}{\sqrt{2}}\times\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)\right)^4 = \left(\frac{1}{\sqrt{2}}\right)^4\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)^4$$

$$= \frac{1}{2^2}\left(\cos(-\pi)+i\sin(-\pi)\right) = -\frac{1}{4}$$

(4) $$\left(\frac{-1+i}{\sqrt{3}+i}\right)^{12} = \left(\frac{\sqrt{2}\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)}{2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)}\right)^{12} = \left(\frac{\sqrt{2}}{2}\right)^{12}\times\left(\frac{\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)}{\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)}\right)^{12}$$

$$= \left(\frac{1}{\sqrt{2}}\right)^{12}\times\left(\cos\left(\frac{7}{12}\pi\right)+i\sin\left(\frac{7}{12}\pi\right)\right)^{12}$$

$$= \frac{1}{2^6}\times\left(\cos(7\pi)+i\sin(7\pi)\right) = -\frac{1}{64}$$

＜ 22 ページ.1 の累乗根 ＞

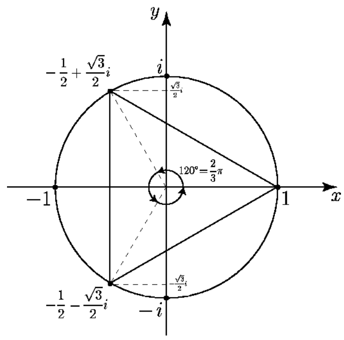
問の解答

(1) $z^3 = 1$

$\cos(3\theta) + i\sin(3\theta) = 1$

$\theta = 0\ ,\ \dfrac{2}{3}\pi\ ,\ \dfrac{4}{3}\pi$

$\underline{z\ =\ 1\ ,\ -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\ ,\ -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i}$

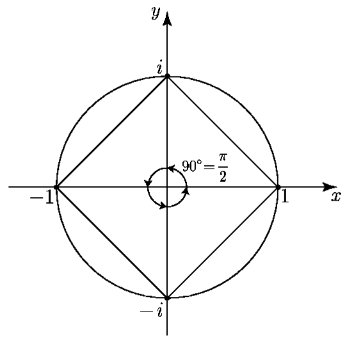


(2) $z^4 = 1$

$\cos(4\theta) + i\sin(4\theta) = 1$

$\theta = 0\ ,\ \dfrac{\pi}{2}\ ,\ \pi\ ,\ \dfrac{3}{2}\pi$

$\underline{z\ =\ \pm 1\ ,\ \pm i}$

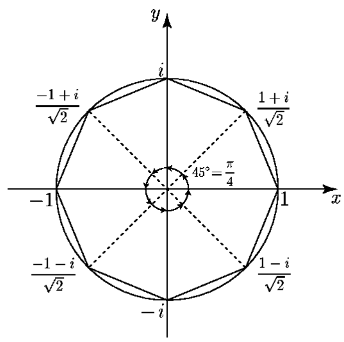


(3) $z^8 = 1$

$\cos(8\theta) + i\sin(8\theta) = 1$

$\theta = 0\ ,\ \dfrac{\pi}{4}\ ,\ \dfrac{\pi}{2}\ ,\ \dfrac{3}{4}\pi\ ,\ \pi\ ,\ \dfrac{5}{4}\pi\ ,\ \dfrac{3}{2}\pi\ ,\ \dfrac{7}{4}\pi$

$\underline{z\ =\ 1\ ,\ \dfrac{1+i}{\sqrt{2}}\ ,\ i\ ,\ \dfrac{-1+i}{\sqrt{2}}\ ,\ -1\ ,\ \dfrac{-1-i}{\sqrt{2}}\ ,\ -i\ ,\ \dfrac{1-i}{\sqrt{2}}}$



＜ 23 ページ. オイラーの公式 1 ＞

問の解答

(1) $e^{2\pi i} = 1$

(2) $e^{-\frac{\pi}{2}i} = -i$

(3) $e^{\frac{3}{4}\pi i} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

(4) $e^{\frac{5}{3}\pi i} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

(5) $e^{-\frac{3}{4}\pi i} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

(6) $e^{-\frac{\pi}{3}i} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

＜ 24 ページ. オイラーの公式 2 ＞

問の解答

(1) $e^{2-2\pi i} = e^2$

(2) $e^{0+\frac{\pi}{3}i} = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

(3) $e^{2+\frac{3}{4}\pi i} = e^2\left(-\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right)$

(4) $e^{\frac{1}{2}-\frac{3}{2}\pi i} = \sqrt{e}\, i$

(5) $e^{\log 2+\frac{5}{4}\pi i} = 2\left(\cos\left(\dfrac{5}{4}\pi\right) + i\sin\left(\dfrac{5}{4}\pi\right)\right) = -\sqrt{2} - \sqrt{2}i$

(6) $e^{\frac{1}{3}\log 8+\frac{\pi}{6}i} = 2\left(\cos\left(\dfrac{\pi}{6}\right) + i\sin\left(\dfrac{\pi}{6}\pi\right)\right) = \sqrt{3} + i$

＜ 25 ページ. 複素数の指数表示 ＞

問 1 の解答

$$e^{i\theta_1} \times e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$$

問 2 の解答

(1) $e^{\frac{3}{2}\pi i} \times e^{\frac{\pi}{2}i} = e^{2\pi i} = 1$

(2) $e^{\frac{4}{3}\pi i} \div e^{\frac{\pi}{6}i} = e^{\frac{7}{6}\pi i} = -\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i$

(3) $\left(e^{\frac{\pi}{8}i}\right)^4 = e^{\frac{\pi}{2}i} = i$

(4) $\left(e^{\frac{\pi}{48}i}\right)^{12} = e^{\frac{\pi}{4}i} = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i$

< 26 ページ. 指数法則 >

問 1 の解答

(2) $\dfrac{e^{z_1}}{e^{z_2}} = e^{\boxed{z_1 - z_2}}$ (3) $\left(e^z\right)^n = e^{\boxed{nz}}$

問 2 の解答

(1) $e^{5+\pi i} \times e^{-1+\pi i} = e^{4+2\pi i} = e^4$

(2) $e^{2+\frac{\pi}{4}i} \div e^{6+\frac{\pi}{4}i} = e^{-4} = \dfrac{1}{e^4}$

(3) $\left(e^{\frac{3}{4}-\frac{3}{8}\pi i}\right)^4 = e^{3-\frac{3}{2}\pi i} = e^3 i$

問 3 の解答

$$\frac{\left(1+i\right)^4}{\left(1+\sqrt{3}i\right)^3} = \frac{\left(\sqrt{2}\right)^4\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^4}{2^3\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3} = \frac{4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^4}{8\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^3} = \frac{\left(\cos\pi+i\sin\pi\right)}{2\left(\cos\pi+i\sin\pi\right)} = \frac{1}{2}$$

＜ 27 ページ. 複素数の簡易表示 ＞

問 1 の解答

(1) $z_1 = \sqrt{3} + i$

$$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= 2e^{\frac{\pi}{6}i}$$

(2) $z_2 = -1 + i$

$$= \sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$$

$$= \sqrt{2}e^{\frac{3}{4}\pi i}$$

(3) $z_3 = -\sqrt{3} - 3i$

$$= 2\sqrt{3}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= 2\sqrt{3}e^{-\frac{2}{3}\pi i}$$

または $\left(2\sqrt{3}e^{\frac{4}{3}\pi i}\right)$

問 2 の解答

(1) $z_1 z_2$

$$= 2e^{\frac{\pi}{6}i} \times \sqrt{2}e^{\frac{3}{4}\pi i}$$

$$= 2\sqrt{2}e^{\frac{11}{12}\pi i}$$

(2) $z_2 z_3$

$$= \sqrt{2}e^{\frac{3}{4}\pi i} \times 2\sqrt{3}e^{-\frac{2}{3}\pi i}$$

$$= 2\sqrt{6}e^{\frac{1}{12}\pi i}$$

$$\left(= 2\sqrt{6}e^{\frac{25}{12}\pi i}\right)$$

(3) $\dfrac{z_3}{z_1} = \dfrac{2\sqrt{3}e^{-\frac{2}{3}\pi i}}{2e^{\frac{\pi}{6}i}}$

$$= \sqrt{3}e^{\left(-\frac{2}{3} - \frac{1}{6}\right)\pi i}$$

$$= \sqrt{3}e^{-\frac{5}{6}\pi i}$$

$$\left(= \sqrt{3}e^{\frac{7}{6}\pi i}\right)$$

＜ 28 ページ. 時間変数 t による微分 1 ＞

問の解答

(1) $\dfrac{d}{dt}\left(9-6t^2+3t^3\right)=-12t+9t^2$

(2) $\dfrac{d}{dt}\left(-t^8+3t^4+2t^2+6e^t\right)=-8t^7+12t^3+4t+6e^t$

(3) $\dfrac{d}{dt}\left(2t^5-6\cos t+\dfrac{1}{2}\log t\right)=10t^4+6\sin t+\dfrac{1}{2t}$

(4) $\dfrac{d}{dt}\left(\dfrac{5}{t}+\dfrac{4}{\sqrt{t^3}}\right)=-\dfrac{5}{t^2}-\dfrac{6}{t^2\sqrt{t}}$

< 29 ページ. 時間変数 t による微分 2 >

問 1 の解答

(1) $\dfrac{d}{dt}\sin(5t+4) = 5\cos(5t+4)$

(2) $\dfrac{d}{dt}e^{3t+2} = 3e^{3t+2}$

(3) $\dfrac{d}{dt}\cos\left(-2t+\dfrac{1}{2}\right) = 2\sin\left(-2t+\dfrac{1}{2}\right) \quad \left(\text{または} = -2\sin\left(2t-\dfrac{1}{2}\right)\right)$

(4) $\dfrac{d}{dt}\log(9-2t) = \dfrac{-2}{9-2t}\left(= \dfrac{2}{2t-9}\right)$

問 2 の解答

(1) $\dfrac{d}{dt}\sin(2t^3-t) = (6t^2-1)\cos(2t^3-t)$

(2) $\dfrac{d}{dt}\left(e^{-t^3}\right) = -3t^2e^{-t^3}$

(3) $\dfrac{d}{dt}\cos(2+3t-4t^2) = (8t-3)\sin(2+3t-4t^2)$

$\left(\text{または} -(8t-3)\sin(4t^2-3t-2)\right)$

(4) $\dfrac{d}{dt}\log(t^5-2t^3+t) = \dfrac{5t^4-6t^2+1}{t^5-2t^3+t}$

＜ 30 ページ. 時間変数 t による微分 3 ＞

問 1 の解答

(1) $\dfrac{d}{dt}(2te^t) = 2e^t + 2te^t$

(2) $\dfrac{d}{dt}(t^3 \cos t) = 3t^2 \cos t - t^3 \sin t$

(3) $\dfrac{d}{dt}\left(\dfrac{1}{2}e^t \sin t\right) = \dfrac{1}{2}e^t \sin t + \dfrac{1}{2}e^t \cos t$

(4) $\dfrac{d}{dt}(t^2 \log t) = 2t \log t + t$

問 2 の解答

(1) $\dfrac{d}{dt}\left(\dfrac{1}{2}e^t \sin(2t)\right) = \dfrac{1}{2}e^t \sin(2t) + e^t \cos(2t)$

(2) $\dfrac{d}{dt}\bigl(e^{3t} \cos(6t)\bigr) = 3e^{3t} \cos(6t) - 6e^{3t} \sin(6t)$

(3) $\dfrac{d}{dt}\bigl(4e^{\frac{t}{2}} \sin(-5t)\bigr) = 2e^{\frac{t}{2}} \sin(-5t) - 20e^{\frac{t}{2}} \cos(-5t)$

$\left(\text{または}\ -2e^{\frac{t}{2}} \sin(5t) - 20e^{\frac{t}{2}} \cos(5t)\right)$

(4) $\dfrac{d}{dt}\bigl(3e^{-2t} \cos(4t)\bigr) = -6e^{-2t} \cos(4t) - 12e^{-2t} \sin(4t)$

＜ 31 ページ. 複素数値関数の微分 1 ＞

問の解答

(1) $z(t) = 3t^2 - 4t + (t^4 + 5t^3)i$

$$\frac{dz}{dt} = 6t - 4 + (4t^3 + 15t^2)i$$

(2) $z(t) = e^{ibt} = \cos(bt) + i\sin(bt)$

$$\frac{dz}{dt} = -b\sin(bt) + bi\cos(bt)$$

(3) $z(t) = e^{(3+2i)t} = e^{3t}\left(\cos(2t) + i\sin(2t)\right) = e^{3t}\cos(2t) + ie^{3t}\sin(2t)$

$$\frac{dz}{dt} = \left\{3e^{3t}\cos(2t) - 2e^{3t}\sin(2t)\right\} + i\left\{3e^{3t}\sin(2t) + 2e^{3t}\cos(2t)\right\}$$

(4) $z(t) = e^{(a+bi)t} = e^{at}\left(\cos(bt) + i\sin(bt)\right) = e^{at}\cos(bt) + ie^{at}\sin(bt)$

$$\frac{dz}{dt} = \left\{ae^{at}\cos(bt) - be^{at}\sin(bt)\right\} + i\left\{ae^{at}\sin(bt) + be^{at}\cos(bt)\right\}$$

＜ 32 ページ. 複素数値関数の微分 2 ＞

問の解答

(1) $\dfrac{d}{dt}e^{3it} = 3ie^{3ti}$

(2) $\dfrac{d}{dt}e^{-2it} = -2ie^{-2ti}$

(3) $\dfrac{d}{dt}e^{bit} = bie^{bit}$

(4) $\dfrac{d}{dt}e^{(1+i)t} = (1+i)e^{(1+i)t}$

(5) $\dfrac{d}{dt}e^{(2-i)t} = (2-i)e^{(2-i)t}$

(6) $\dfrac{d}{dt}e^{(-3+2i)t} = (-3+2i)e^{(-3+2i)t}$

(7) $\dfrac{d}{dt}e^{(a-i)t} = (a-i)e^{(a-i)t}$

(8) $\dfrac{d}{dt}e^{(a-bi)t} = (a-bi)e^{(a-bi)t}$

＜ 33 ページ. 複素数の練習 1 ＞

問 1 の解答

(1) $i^7 + i^4 + i = -i + 1 + i = 1$

(2) $(i+1)(i^2 - i + 1) = i^3 + 1 = -i + 1$

(3) $\left(\dfrac{1+i^3}{2}\right)\left(\dfrac{1-i^3}{2}\right) = \left(\dfrac{1-i}{2}\right)\left(\dfrac{1+i}{2}\right) = \dfrac{1^2 - i^2}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

(4) $\dfrac{1-i}{1+i} = \dfrac{(1-i)^2}{1^2 - i^2} = \dfrac{1 - 2i + i^2}{1+1} = \dfrac{-2i}{2} = -i$

(5) $\dfrac{2}{i - \sqrt{3}} = \dfrac{2\left(i + \sqrt{3}\right)}{i^2 - 3} = \dfrac{2\left(i + \sqrt{3}\right)}{4} = -\dfrac{\sqrt{3} + i}{2}$

(6) $\sqrt{-10} \times \sqrt{-6} \div \sqrt{-105} \times \sqrt{-7} = \dfrac{\sqrt{10}\, i \times \sqrt{6}\, i \times \sqrt{7}\, i}{\sqrt{105}\, i} = -\sqrt{\dfrac{420}{105}} = -2$

問 2 の解答

$$x = \frac{1 \pm \sqrt{1 - 24}}{4} = \frac{1 \pm \sqrt{23} i}{4}$$

問 3 の解答

$\overline{z} = 3 - 4i$

$z\overline{z} = 3^2 + 4^2 = 25$

$|z| = \sqrt{25} = 5$

$z^2 = (3 + 4i)^2 = 9 + 24i + 16i^2 = -7 + 24i$

問 4 の解答

(1) $3 - \sqrt{3}i = 2\sqrt{3}\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right)$

$= 2\sqrt{3}\left(\cos\left(-\dfrac{\pi}{6}\right) + i\sin\left(-\dfrac{\pi}{6}\right)\right)$

$\left(= 2\sqrt{3}\left(\cos\left(-\dfrac{11}{6}\pi\right) + i\sin\left(-\dfrac{11}{6}\pi\right)\right)\right)$

(2) $-2 + 2i = 2\sqrt{2}\left(-\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}i}\right)$

$= 2\sqrt{2}\left(\cos\left(\dfrac{3}{4}\pi\right) + i\sin\left(\dfrac{3}{4}\pi\right)\right)$

問 5 の解答

(1) $\left(\dfrac{\sqrt{3} + i}{2}\right)^{12} = \left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)^{12} = \cos(2\pi) + i\sin(2\pi) = 1$

(2) $(1 - i)^8 = \left(\sqrt{2}\left(\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}i\right)\right)^8 = \left(\sqrt{2}\right)^8\left(\cos\left(-\dfrac{\pi}{4}\right) + i\sin\left(-\dfrac{\pi}{4}\right)\right)^8$

$= 2^4\left(\cos\left(-2\pi\right) + i\sin\left(-2\pi\right)\right) = 16$

＜ 34 ページ. 複素数の練習 2 ＞

問 1 の解答

(1) $e^{-\frac{2\pi}{3}i} = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

(2) $e^{3+\frac{\pi}{4}i} = e^3\left(\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right)$

(3) $e^{\frac{\pi}{3}i} \div e^{\frac{\pi}{2}i} = e^{-\frac{\pi}{6}i} = \dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i$

問 2 の解答

(1) $\dfrac{1-\sqrt{3}i}{2} = e^{-\frac{\pi}{3}i}$ $\left(\text{または}\ \ e^{\frac{5}{3}\pi i}\right)$

(2) $-\dfrac{\sqrt{2}e}{2} + \dfrac{\sqrt{2}e}{2}i = e^{1+\frac{3}{4}\pi i}$

問 3 の解答

(1) $1+\sqrt{3}i = 2e^{\frac{\pi}{3}i}$

(2) $-3+\sqrt{3}i = 2\sqrt{3}\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = 2\sqrt{3}e^{\frac{5\pi}{6}i}$

問 4 の解答

(1) $e^{\frac{5\pi}{3}i}\cos\left(\dfrac{5}{3}\pi\right) + i\sin\left(\dfrac{5}{3}\pi\right)$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

(2) $e^{\frac{2+3\pi i}{4}} = e^{\frac{1}{2}} \times e^{\frac{3}{4}\pi i}$

$$= \sqrt{e}\left\{-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right\} \quad \left(= \frac{\sqrt{2e}}{2}(-1+i)\right)$$

(3) $\left(e^{\frac{\pi}{6}i}\right)^7 \div e^{\frac{4\pi}{3}i} = e^{\left(\frac{7}{6}\pi - \frac{4}{3}\pi\right)i} = e^{-\frac{\pi}{6}i} = \dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i$

(4) $\left(\dfrac{1-i}{2}\right)e^{(3+5i)t} + \left(\dfrac{1+i}{2}\right)e^{(3-5i)t}$

$$= \left(\frac{1-i}{2}\right)e^{3t}\left(\cos(5t) + i\sin(5t)\right) + \left(\frac{1+i}{2}\right)e^{3t}\left(\cos(5t) - i\sin(5t)\right)$$

$$= \frac{e^{3t}}{2}\left\{\left(\cos(5t)+\sin(5t)\right) + i\left(-\cos(5t)+\sin(5t)\right) + \left(\cos(5t)+\sin(5t)\right) + i\left(\cos(5t)-\sin(5t)\right)\right\}$$

$$= e^{3t}\left\{\cos(5t) + \sin(5t)\right\}$$

問 5 の解答

$\cos\theta = \dfrac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$　　$\sin\theta = \dfrac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$ $\left(= \dfrac{i}{2}\left(e^{-i\theta} - e^{i\theta}\right)\right)$

問 6 の解答

(1) $\dfrac{d}{dt}e^{t^2+t} = (2t+1)e^{t^2+t}$

(2) $\dfrac{d}{dt}\{e^{2t}\cos(3t)\} = 2e^{2t}\cos(3t) - 3e^{2t}\sin(3t)$

(3) $\dfrac{d}{dt}e^{-3ti} = -3ie^{-3ti}$

(4) $\dfrac{d}{dt}e^{(4+5i)t} = (4+5i)e^{(4+5i)t}$

＜ 35ページ. 微分方程式 ＞

問の解答

(1) $\dfrac{dy}{dt} = 2y$

1階微分方程式

(2) $\dfrac{d^2y}{dt^2} = -9y$

2階微分方程式

(3) $\dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} + t^4 = 0$

3階微分方程式

＜ 36 ページ. 微分方程式の解 1 ＞

問の解答

$y = 3e^t$

($y = -e^t$ など)

＜ 37 ページ. 微分方程式の解 2 ＞

問の解答

(1) $t = 0$ のとき $y = 3$

$C = 3$　　　$\underline{y = 3e^t}$

(2) $t = 0$ のとき $y = -2$

$C = -2$　　　$\underline{y = -2e^t}$

(3) $t = 0$ のとき $y = 0$

$C = 0$　　　$\underline{y = 0}$

＜ 38 ページ. 微分方程式の解 3 ＞

問 1 の解答

$t = 0$ のとき $y = 2$

問 2 の解答

$y = 3e^{-t}$, $(y = -e^{-t}$など$)$

問 3 の解答

$y = Ce^{-t}$

＜ 39 ページ. 積分の復習 ＞

問の解答

(1) $\displaystyle\int e^y \frac{dy}{dt}dt = \int e^y dy = e^y + C$

(2) $\displaystyle\int \frac{1}{y^2}\frac{dy}{dt}dt = \int \frac{1}{y^2}dy = -\frac{1}{y} + C$

(3) $\displaystyle\int \sin y \frac{dy}{dt}dt = \int \sin y dy = -\cos y + C$

(4) $\displaystyle\int \cos y \frac{dy}{dt}dt = \int \cos y dy = \sin y + C$

＜ 40 ページ. 求積法 ＞

問の解答

(1) $\dfrac{dy}{dt} = 3t + 6$

$\underline{y = \dfrac{3}{2}t^2 + 6t + C}$

(2) $\dfrac{dy}{dt} = \dfrac{1}{2}t^3 + 5t^4$

$\underline{y = \dfrac{1}{8}t^4 + t^5 + C}$

(3) $\dfrac{dy}{dt} = -\dfrac{2}{t^2} + \dfrac{1}{t}$

$\underline{y = \dfrac{2}{t} + \log t + C}$

(4) $\dfrac{dy}{dt} = 4\sin t - 5\cos t$

$\underline{y = -4\cos t - 5\sin t + C}$