

高知工科大学

基礎数学ワークブック

(2001年度版)

番外編 2

「データの関数近似」

解答

## ＜ 番外編 2 「データの関数近似」 解答 ＞

### p.2 < ラグランジュ補間 1 >

$$\begin{aligned} \text{問 1} \quad y &= \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0 = \frac{y_1(x - x_0) - y_0(x - x_0) + y_0(x_1 - x_0)}{(x_1 - x_0)} \\ &= \frac{x - x_1}{x_0 - x_1} \times y_0 + \frac{x - x_0}{x_1 - x_0} \times y_1 \end{aligned}$$

$$\text{問 2} \quad p_1(x_0) = \frac{x_0 - x_1}{x_0 - x_1} \times y_0 + \frac{x_0 - x_0}{x_1 - x_0} \times y_1 = y_0$$

$$p_1(x_1) = \frac{x_1 - x_1}{x_0 - x_1} \times y_0 + \frac{x_1 - x_0}{x_1 - x_0} \times y_1 = y_1$$

$$\text{問 3} \quad p_2(x_0) = y_0, \quad p_2(x_1) = y_1, \quad p_2(x_2) = y_2$$

$$\begin{aligned} \text{問 4} \quad p_2(x) &= \frac{(x-3)(x-4)}{(1-3)(1-4)} \times 1 + \frac{(x-1)(x-4)}{(3-1)(3-4)} \times (-1) + \frac{(x-1)(x-3)}{(4-1)(4-3)} \times 4 \\ &= 2x^2 - 9x + 8 \end{aligned}$$

$$\text{(答)} \quad \underline{y = 2x^2 - 9x + 8}$$

### p.3 < ラグランジュ補間 2 >

$$\text{問 1} \quad p_3(x_0) = y_0, \quad p_3(x_1) = y_1, \quad p_3(x_2) = y_2, \quad p_3(x_3) = y_3$$

$$\text{問 2} \quad p_4(x_0) = y_0, \quad p_4(x_1) = y_1, \quad p_4(x_2) = y_2, \quad p_4(x_3) = y_3, \quad p_4(x_4) = y_4$$

### p.4 < ラグランジュ補間 3 >

$$p_4(x) = \sum_{k=0}^4 \frac{\prod_{\substack{0 \leq i \leq 4 \\ i \neq k}} (x - x_i)}{\prod_{\substack{0 \leq i \leq 4 \\ i \neq k}} (x_k - x_i)} \times y_k$$

### p.6 < 片対数方眼紙 2 >

$$\text{問 1} \quad y = 30 \times 10^{-\frac{1}{5}x} \qquad y = 2 \times 10^{\frac{1}{3}(x-1)}$$

$$\text{問 2} \quad y = 2e^{\frac{2.3}{3}(x-1)} \qquad y = 30e^{-\frac{2.3}{5}x}$$

## ＜ 番外編 2 「データの関数近似」 解答 ＞

### p.8 < 両対数方眼紙 2 >

$$y = 10x \qquad y = \frac{1}{100}x^2$$

$$y = 2\sqrt{x} \qquad y = \frac{100}{x}$$

### p.10 < 三角多項式補間 2 >

$$(1) f_4(0) = 1 - \cos 0 + 2 \sin 0 + 3 \cos 0 = 1 - 1 + 0 + 3 = 3$$

$$(2) f_4\left(\frac{\pi}{2}\right) = 1 - \cos\left(\frac{\pi}{2}\right) + 2 \sin\left(\frac{\pi}{2}\right) + 3 \cos\left(\frac{\pi}{2}\right) = 1 - 0 + 2 - 3 = 0$$

$$(3) f_4(\pi) = 1 - \cos \pi + 2 \sin \pi + 3 \cos \pi = 1 - (-1) + 0 + 3 = 5$$

$$(4) f_4\left(\frac{3}{2}\pi\right) = 1 - \cos\left(\frac{3}{2}\pi\right) + 2 \sin\left(\frac{3}{2}\pi\right) + 3 \cos\left(\frac{3}{2}\pi\right) = 1 - 0 - 2 - 3 = -4$$

### p.11 < 三角多項式補間 3 >

$$A_2 = \frac{1}{8} \sum_{\ell=0}^7 y_\ell \cos(2x_\ell) = \frac{1}{8} \left\{ 0 + 3 \times 0 + 0 - 3 \times 0 + 0 - 0 + 0 + 0 \right\} = 0$$

$$A_4 = \frac{1}{8} \sum_{\ell=0}^7 y_\ell \cos(4x_\ell) = \frac{1}{8} \left\{ 0 + 3 \times (-1) + 0 - 3 \times (-1) + 0 - (-1) + 0 + (-1) \right\} = 0$$

$$B_1 = \frac{1}{8} \sum_{\ell=0}^7 y_\ell \sin(x_\ell) = \frac{1}{8} \left\{ 0 + 3 \times \frac{\sqrt{2}}{2} + 0 - 3 \times \frac{\sqrt{2}}{2} + 0 - \left(-\frac{\sqrt{2}}{2}\right) + 0 + \left(-\frac{\sqrt{2}}{2}\right) \right\} = 0$$

$$B_2 = \frac{1}{8} \sum_{\ell=0}^7 y_\ell \sin(2x_\ell) = \frac{1}{8} \left\{ 0 + 3 \times 1 + 0 - 3 \times (-1) + 0 - 1 + 0 + (-1) \right\} = \frac{1}{2}$$

$$B_3 = \frac{1}{8} \sum_{\ell=0}^7 y_\ell \sin(3x_\ell) = \frac{1}{8} \left\{ 0 + 3 \times \frac{\sqrt{2}}{2} + 0 - 3 \times \frac{\sqrt{2}}{2} + 0 - \left(-\frac{\sqrt{2}}{2}\right) + 0 + \left(-\frac{\sqrt{2}}{2}\right) \right\} = 0$$

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### p.12 < 三角多項式補間 4 >

$$(1) \quad f_8(x_4) = f_8(\pi) = \sqrt{2} \cos \pi + \sin(2\pi) - \sqrt{2} \cos(3\pi) \\ = \sqrt{2} \times (-1) + 0 - \sqrt{2} \times (-1) = 0$$

$$(2) \quad f_8(x_5) = f_8\left(\frac{5\pi}{4}\right) = \sqrt{2} \cos\left(\frac{5}{4}\pi\right) + \sin\left(\frac{5}{2}\pi\right) - \sqrt{2} \cos\left(\frac{15}{4}\pi\right) \\ = \sqrt{2} \times \left(-\frac{\sqrt{2}}{2}\right) + 1 - \sqrt{2} \times \frac{\sqrt{2}}{2} = -1$$

$$(3) \quad f_8(x_6) = f_8\left(\frac{3\pi}{2}\right) = \sqrt{2} \cos\left(\frac{3}{2}\pi\right) + \sin(3\pi) - \sqrt{2} \cos\left(\frac{9}{2}\pi\right) \\ = \sqrt{2} \times 0 + 0 - \sqrt{2} \times 0 = 0$$

$$(4) \quad f_8(x_7) = f_8\left(\frac{7\pi}{4}\right) = \sqrt{2} \cos\left(\frac{7}{4}\pi\right) + \sin\left(\frac{7}{2}\pi\right) - \sqrt{2} \cos\left(\frac{21}{4}\pi\right) \\ = \sqrt{2} \times \frac{\sqrt{2}}{2} + (-1) - \sqrt{2} \times \left(-\frac{\sqrt{2}}{2}\right) = 1$$

### p.17 < 三角多項式補間 9 >

$$\text{問 1} \quad B_k = \frac{1}{N} \sum_{\ell=0}^{N-1} y_\ell \sin\left(\frac{2\pi k\ell}{N}\right) = \frac{1}{N} \sum_{\ell=0}^{N-1} y_\ell \left\{ -\sin\left(\frac{2\pi(N-k)\ell}{N}\right) \right\} \\ = -\frac{1}{N} \sum_{\ell=0}^{N-1} y_\ell \sin\left(\frac{2\pi(N-k)\ell}{N}\right) = -B_{N-k}$$

$$\text{問 2} \quad B_n = \frac{1}{2n} \sum_{\ell=0}^{2n-1} y_\ell \sin\left(\frac{\pi n\ell}{n}\right) = \frac{1}{2n} \sum_{\ell=0}^{2n-1} y_\ell \sin(\pi\ell) = 0$$

## ＜ 番外編 2 「データの関数近似」 解答 ＞

### p.20 < 離散フーリエ変換 1 >

$e^{\frac{\pi(2n-k)\ell}{n}i} = e^{-\frac{\pi k\ell}{n}i}$  とオイラーの公式及び (2) 式より

$$\begin{aligned}
 y_\ell &= A_0 + 2 \sum_{k=1}^{n-1} \left\{ A_k \cos\left(\frac{\pi k\ell}{n}\right) + B_k \sin\left(\frac{\pi k\ell}{n}\right) \right\} + A_n \cos(\pi\ell) \\
 &= A_0 + \sum_{k=1}^{n-1} \left\{ (A_k - B_k i) e^{\frac{\pi k\ell}{n}i} + (A_k + B_k i) e^{-\frac{\pi k\ell}{n}i} \right\} + A_n \cos(\pi\ell) \\
 &= (A_0 - B_0 i) e^0 + \sum_{k=1}^{n-1} (A_k - B_k i) e^{\frac{\pi k\ell}{n}i} \\
 &\quad + \sum_{k=1}^{n-1} (A_{2n-k} - B_{2n-k} i) e^{\frac{\pi(2n-k)\ell}{n}i} + (A_n - B_n i) e^{\frac{\pi n\ell}{n}i} \\
 &= \sum_{k=0}^{2n-1} (A_k - B_k i) e^{\frac{\pi k\ell}{n}i}
 \end{aligned}$$

(注) ここで和の順序を変えた等式

$$\sum_{k=1}^{n-1} (A_{2n-k} - B_{2n-k} i) e^{\frac{\pi(2n-k)\ell}{n}i} = \sum_{k=n+1}^{2n-1} (A_k - B_k i) e^{\frac{\pi k\ell}{n}i} \quad \text{を使った。}$$

### p.22 < 離散フーリエ変換 3 >

$$\begin{aligned}
 C_{N-k} &= \frac{1}{N} \sum_{\ell=0}^{N-1} y_\ell e^{-\frac{2\pi(N-k)\ell}{N}i} \\
 &= \frac{1}{N} \sum_{\ell=0}^{N-1} y_\ell e^{-2\pi\ell i + \frac{2\pi k\ell}{N}i} \\
 &= \frac{1}{N} \sum_{\ell=0}^{N-1} y_\ell e^{\frac{2\pi k\ell}{N}i} \\
 &= \overline{\frac{1}{N} \sum_{\ell=0}^{N-1} y_\ell e^{-\frac{2\pi k\ell}{N}i}} = \overline{C_k}
 \end{aligned}$$

## < 番外編 2 「データの関数近似」 解答 >

### p.26 < 回帰直線 3 >

$$\begin{aligned}
 y &= \frac{S_{xy}}{S_{xx}}(x - \bar{x}) + \bar{y} \\
 &= \frac{5.65}{7.41}(x - 37.7) + 79.5
 \end{aligned}$$

